ECE 391 – TRANSMISSION LINES AND ELECTROMAGNETIC WAVES

Spring Term 2003

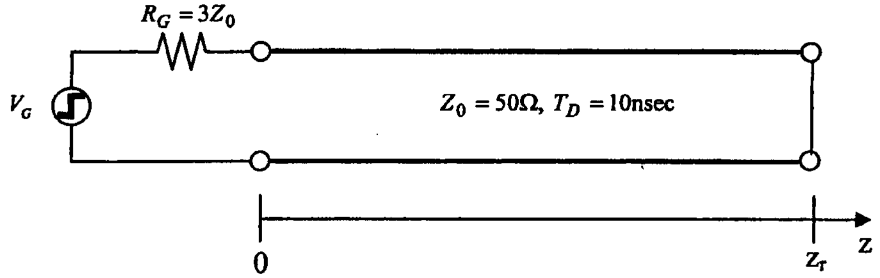
Midterm I

Name: Solutions SSN: xxx-xx-__________

(Last name, first name)

Exam is closed book, closed notes; ***one*** *sheet (2 pages) of formulas allowed; 50 minutes. Show all your work on the pages provided. No extra pages (use back if necessary). Read each question very carefully. Maximum number of points for each problem is given in parenthesis. Total: 40 points.*

1. (25 pts.) At time $t = 0$, a generator with internal resistance $R_G = 3Z_0$ and $V_G = 4$ V is connected to a lossless transmission line with characteristic impedance $Z_0 = 50\Omega$ and delay time $T_D = 10$ nsec. The transmission line is short-circuited at the far end. The propagation velocity on the transmission line is 3×10^8 m/s.



(a) Determine the length of the line in meters.

$$z_T = V_p \cdot T_D = 3\times10^8 \tfrac{m}{s} \times 10\times10^{-9} s = \boxed{3m}$$

(b) Determine the voltage reflection coefficients at both ends.

$$\boxed{\rho_T = -1} \text{ (short circuit)}$$

$$\rho_G = \frac{3Z_0 - Z_0}{3Z_0 + Z_0} = \boxed{\frac{1}{2}}$$

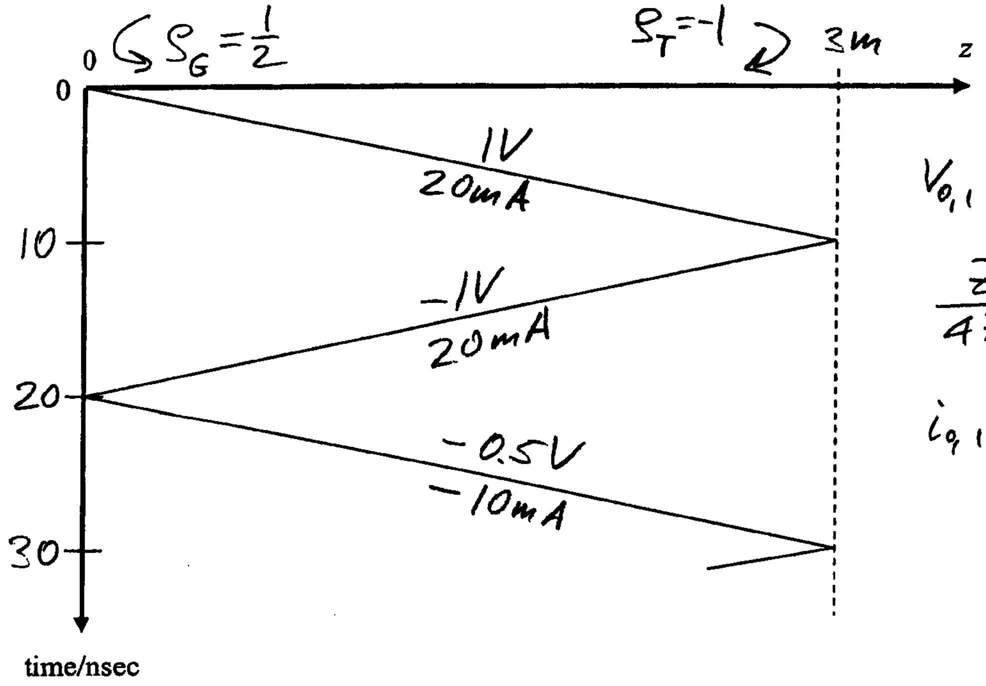
(c) Determine the distributed capacitance and inductance parameters of the transmission line.

$$V_p = \frac{1}{\sqrt{\ell c}} \qquad Z_0 = \sqrt{\frac{\ell}{c}} \qquad V_p = \frac{1}{\sqrt{\ell c}} \qquad Z_0 \sqrt{\ell/c}$$

$$\Rightarrow \ell = \frac{Z_0}{V_p} = 166.7\,nH/m \qquad \ell = \frac{Z_0}{V_p} \Rightarrow \frac{50}{3.0\times10^8}$$

$$c = \frac{1}{Z_0 V_p} = 66.7\,pF/m \qquad C = \frac{1}{Z_0 V_p} =$$

(d) Fill in the <u>numerical</u> **voltage** and **current** values for the first **three** wave components and add the time scale and length scale in the lattice diagram shown below.

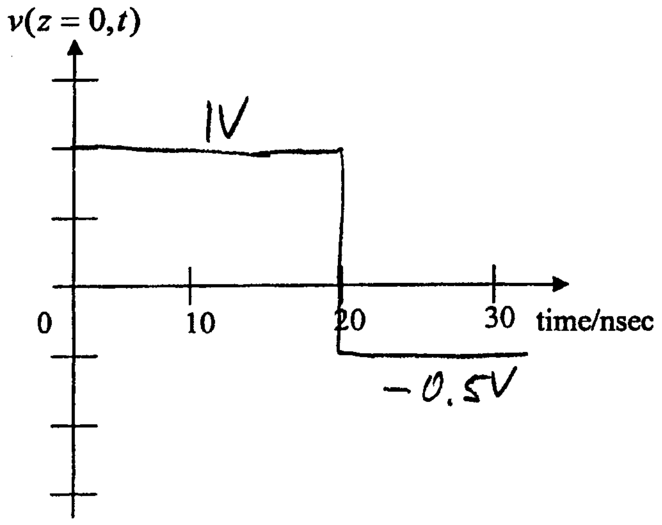


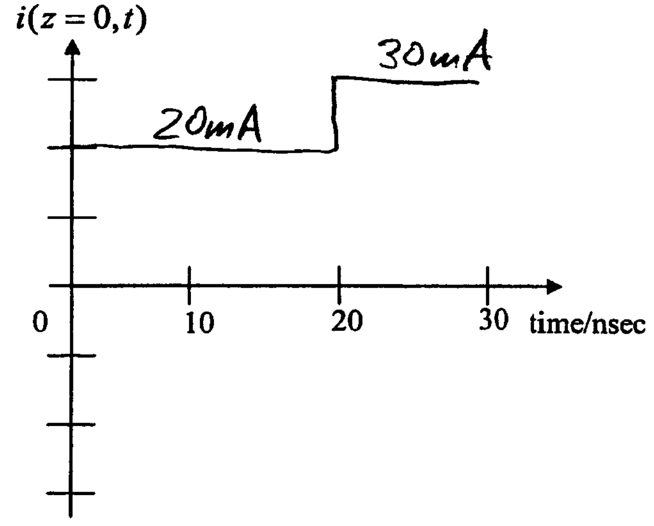
$$V_{0,1} = V_G' = \frac{Z_0}{R_G+Z_0} V_G =$$

$$\frac{Z_0}{4Z_0} V_G = \boxed{1V}$$

$$i_{0,1} = \frac{V_{0,1}}{Z_0} = 20mA$$

(e) Sketch voltage $v(z=0,t)$ and current $i(z=0,t)$ at the beginning of the line (z=0) for $0 \le t \le 30$ nsec.



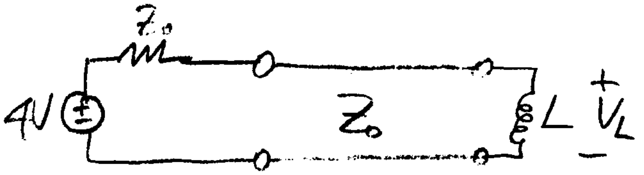


(f) Determine $v(z=0)$ and $i(z=0)$ at the input of the line for $t \to \infty$.

$V_\infty = 0$ (short circuit)

$i_\infty = \frac{V_G}{R_G} = \frac{4V}{150\Omega} = \boxed{26.7mA}$

(g) Now, the source resistance is changed to $R_G = Z_0 = 50\Omega$ (matched), and the short circuit is replaced by an inductor with inductance $L = 250\,\text{nH}$. Determine and sketch the voltage $v(z=0,t)$ at the input of the line if the source is turned on at time $t = 0$ and no current or voltage exist on the line for $t < 0$.



$V_G' = \frac{1}{2} V_G = 2V$

at the inductor: $V_L(t) = 2V_{0,1} e^{-\frac{t-TD}{\tau}} u(t-TD)$

$\tau = \frac{L}{Z_0} = \boxed{5ns}$

returning wave: $V_{r,1}(t, z=z_r) = V_L(t) - V_{0,1}(t, z=z_r)$

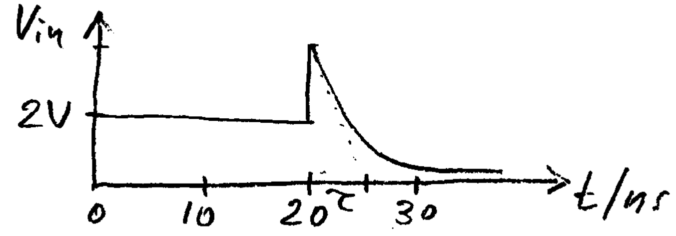
$= \left(2e^{-\frac{t-TD}{\tau}} - 1\right) V_G' u(t-TD)$

at the input:

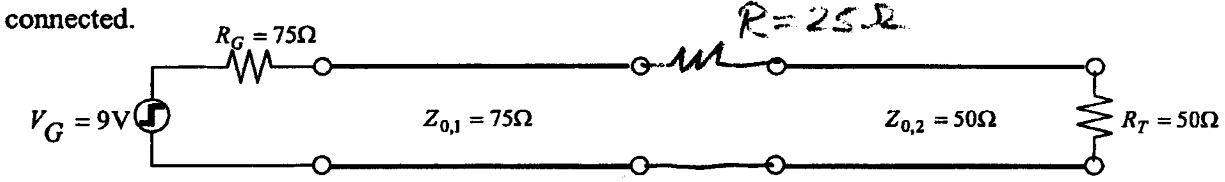
$V_{r,1}(t, z=0) = \left[2e^{-\frac{t-2TD}{\tau}} - 1\right] V_G' u(t-2TD)$

total voltage:

$V_{in}(t) = V_{0,1}(t, z=0) + V_{r,1}(t, z=0) = V_{0,1} u(t) + \left[2e^{-\frac{t-2TD}{\tau}} - 1\right] V_G' u(t-2TD)$



2. (15 pts.) Two lossless transmission lines with different characteristic impedances are to be connected.



(a) You have available one resistor R with resistance value of your choice. Show in the figure above how you would connect the two lines and your resistor to eliminate any reflected waves on the two lines. Determine R.

lines are matched at generator side and termination

$$\Rightarrow \rho_{11} \overset{!}{=} 0 \Rightarrow \text{add series resistor } R$$

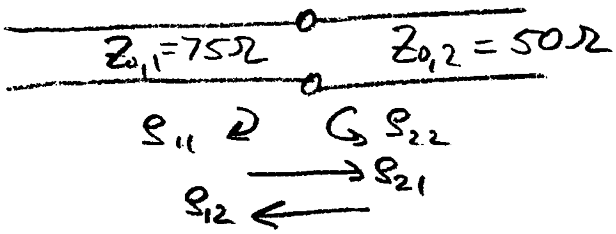
$$R + Z_{0,2} = Z_{0,1} \Rightarrow \boxed{R = 25\Omega}$$

(b) Determine the steady-state voltages at the input of the first line (source side) and at the termination of the second line (R_T) (i.e. for $t \to \infty$) for the configuration determined in part a.

source side: $$V_\infty = \frac{R + R_T}{R_G + R + R_T} V_G = \boxed{4.5V}$$

termination: $$V_\infty = \frac{R_T}{R_G + R + R_T} V_G = \boxed{3V}$$

(c) Determine the reflection and transmission coefficients ρ_{11}, ρ_{22}, ρ_{12}, ρ_{21} at the junction of the two lines if the lines are directly connected without the additional resistor.



$$\rho_{11} = \frac{Z_{0,2} - Z_{0,1}}{Z_{0,2} + Z_{0,1}} = \boxed{-\frac{1}{5} = -0.2}$$

$$\rho_{22} = -\rho_{11} = \boxed{+0.2}$$

$$\rho_{12} = 1 + \rho_{22} = \boxed{1.2}$$

$$\rho_{21} = 1 + \rho_{11} = \boxed{0.8}$$

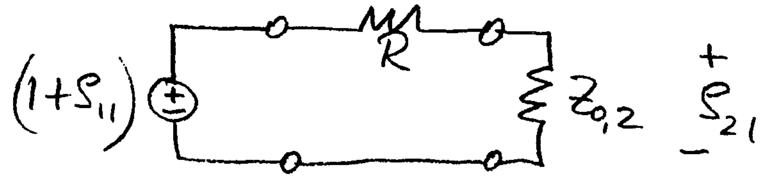
(d) Determine the two reflection coefficients, ρ_{11} and ρ_{22}, at the junction of the two lines <u>when the lines are connected as determined in part a.</u>

$\boxed{\rho_{11} = 0}$ (as required in part a)

$$\rho_{22} = \frac{Z_{0,1} + R - Z_{0,2}}{Z_{0,1} + R + Z_{0,2}} = \frac{100 - 50}{100 + 50} = \boxed{\frac{1}{3}}$$

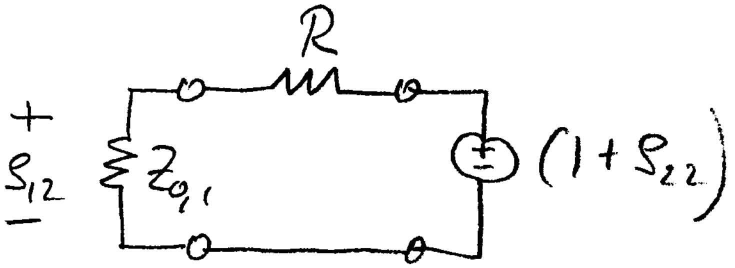
(e) Determine the two transmission coefficients, ρ_{12} and ρ_{21}, at the junction of the two lines <u>when the lines are connected as determined in part a.</u>

ρ_{21}:



$$\rho_{21} = \frac{Z_{0,2}}{R + Z_{0,2}}(1 + \rho_{11}) = \frac{Z_{0,2}}{R + Z_{0,2}} = \boxed{0.666 \ \left(\frac{2}{3}\right)}$$

ρ_{12}:



$$\rho_{12} = \frac{Z_{0,1}}{R + Z_{0,1}}(1 + \rho_{22}) = \frac{75}{100}\ \frac{4}{3} = \boxed{1}$$

ECE 391 – TRANSMISSION LINES AND ELECTROMAGNETIC WAVES

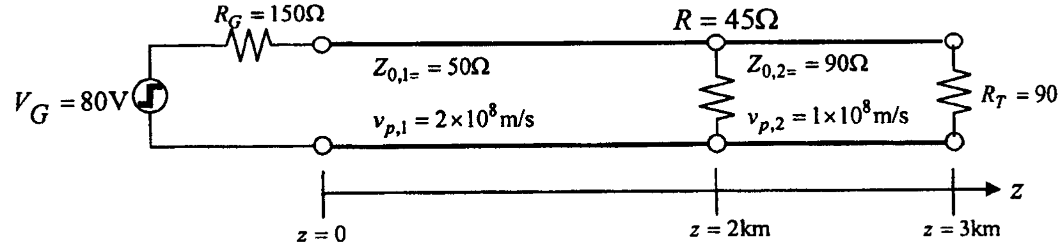
Winter Term 2004

Midterm I

Name: Solutions SSN: xxx-xx-__________
(Last name, first name)

Exam is closed book, closed notes; ***one*** *sheet (2 pages) of formulas allowed; 50 minutes. Show all your work on the pages provided. No extra pages (use back if necessary). Read each question very carefully. Maximum number of points for each problem is given in parenthesis. Total: 40 points.*

1. (20 pts.) Two lossless transmission lines with different characteristic impedances are connected in tandem and excited by a step voltage source as shown below.



(a) Determine the delay times of the two transmission lines.

$$TD_1 = \frac{2Km}{v_{p,1}} = \frac{2\times10^3 m}{2\times10^8 \frac{m}{s}} = 1\times10^{-5} s = \boxed{10\mu sec}$$

$$TD_2 = \frac{1Km}{v_{p,2}} = \frac{1\times10^3 m}{1\times10^8 \frac{m}{s}} = \boxed{10\mu sec}$$

(b) Determine the voltage and current of the first outgoing wave component traveling on the transmission line connected to the source.

$$V_{0,1}^I = \frac{Z_{0,1}}{R_G + Z_{0,1}} V_G = \frac{50}{150+50} 80V = \boxed{20V}$$

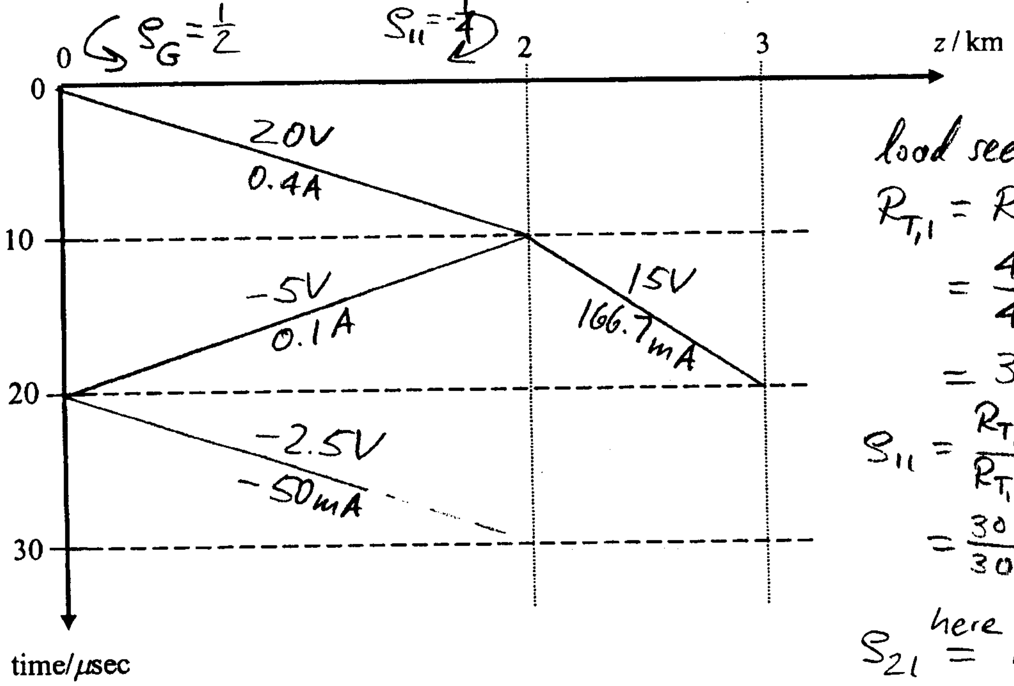
$$i_{0,1}^I = \frac{V_{0,1}^I}{Z_{0,1}} = \frac{20V}{50\Omega} = \boxed{400mA}$$

(c) Determine the reflection coefficient at the source and the reflection coefficient at the termination of the second line.

$$\rho_G = \frac{R_G - Z_{0,1}}{R_G + Z_{0,1}} = \frac{150-50}{150+50} = \boxed{\frac{1}{2}}$$

$$\rho_T = \frac{R_T - Z_{0,2}}{R_T + Z_{0,2}} = \frac{90-90}{90+90} = \boxed{0}$$

(d) Complete the lattice diagram for the system shown below for $0 \le t \le 25\mu\text{sec}$. Give explicit numerical values for the voltage and current of each wave component.



load seen by line I

$R_{T,1} = R // Z_{0,2} = \frac{45\Omega \cdot 90\Omega}{45\Omega + 90\Omega} = 30\Omega$

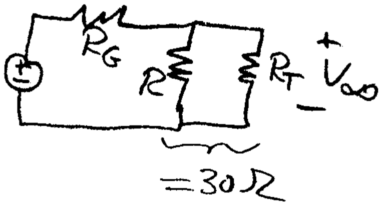
$\rho_{11} = \frac{R_{T,1} - Z_{01}}{R_{T1} + Z_{0,1}} = \frac{30-50}{30+50} = \boxed{-\frac{1}{4}}$

$\rho_{21} \overset{here}{=} 1 + \rho_{11} = \boxed{\frac{3}{4}}$

(e) Determine the steady-state voltages (i.e. for $t \to \infty$) at the input of the left cable ($z = 0$), at the junction between the two cables ($z = 2\text{km}$), and at the end of the second (shorter) cable ($z = 3\text{km}$). What is the steady-state current through the load resistor $R_T = 90\Omega$?

for $t \to \infty$, $V(z=0) = V(z=2km) = V(z=3km) = V_\infty$

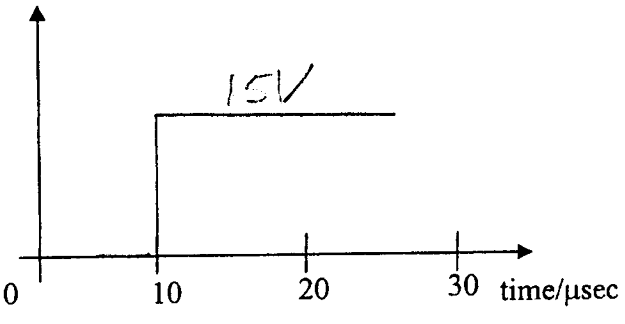
$V_\infty = \frac{30\Omega}{R_G + 30\Omega} V_G$



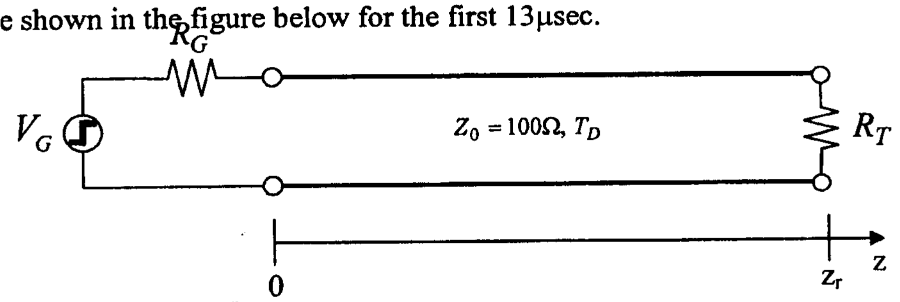
$= \frac{30}{180} 80V = \boxed{13.3V}$

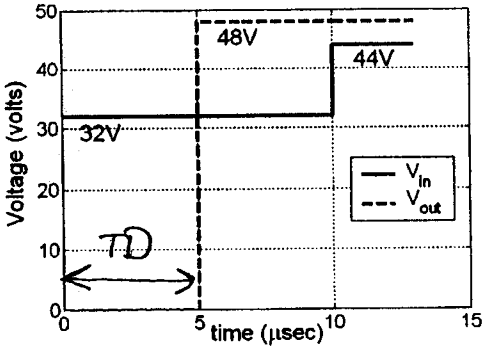
$i_\infty|_{R_T} = \frac{V_\infty}{R_T} = \boxed{148.1mA}$

(f) Sketch the voltage across the resistor $R = 45\Omega$ at the junction of the two lines for $0 \le t \le 25\mu\text{sec}$ and specify numerical voltage values as appropriate.



2. (20 pts.) At time $t = 0$, a step-function generator with internal resistance R_G and open-circuit voltage V_G is connected to a lossless transmission line of length $z_r = 500\text{m}$ having characteristic impedance $Z_0 = 100\Omega$. The transmission line is terminated in a resistance R_T at the far end. The voltage waveforms at the input of the line ($z = 0$) and at the end of the line ($z = z_r$) are shown in the figure below for the first 13μsec.





(a) Determine the delay time *TD* of the transmission line and the velocity of propagation on the line.

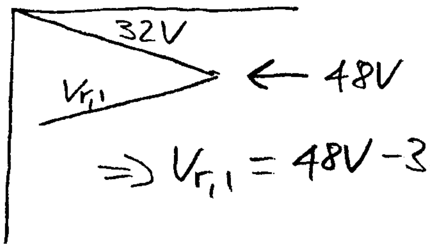
$$TD = \boxed{5\mu sec} \qquad V_p = \frac{z_r}{TD} = \frac{500m}{5\times10^{-6}s} = \boxed{1\times10^{8}\frac{m}{s}}$$

(b) Determine the voltage and the current of the first outgoing wave component.

$$V_{0,1} = \boxed{32V}$$

$$i_{0,1} = \frac{V_{0,1}}{Z_0} = \boxed{0.32A}$$

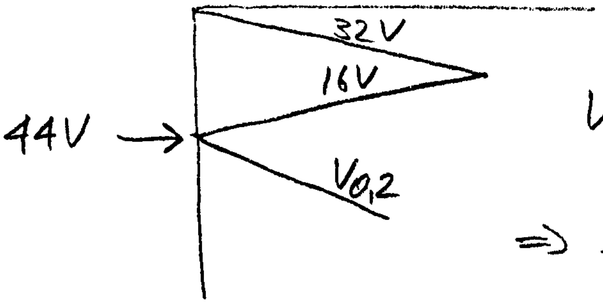
(c) Determine the reflection coefficient ρ_T at the terminating load. (Hint: sketch a lattice diagram.)



$$\Rightarrow V_{r,1} = 48V - 32V = 16V$$

$$\rho_T = \frac{V_{r,1}}{V_{0,1}} = \frac{16V}{32V} = \boxed{0.5}$$

(d) Determine the reflection coefficient ρ_G at the generator side. (Hint: sketch a lattice diagram.)



$$V_{0,2} = 44V - 32V - 16V = -4V$$

$$\Rightarrow \rho_G = \frac{V_{0,2}}{V_{r,1}} = \frac{-4V}{16V} = \boxed{-\frac{1}{4}}$$

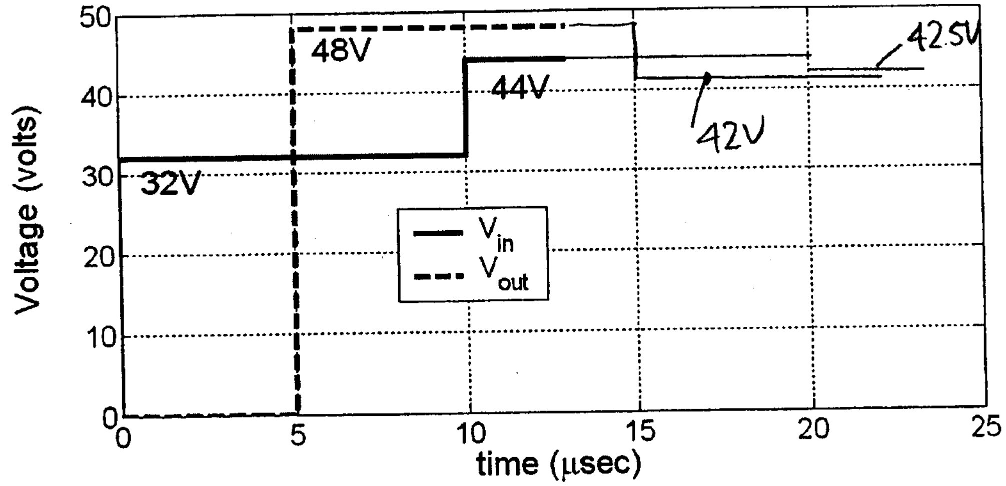
(e) Determine generator resistance R_G, load resistance R_T, and generator voltage V_G.

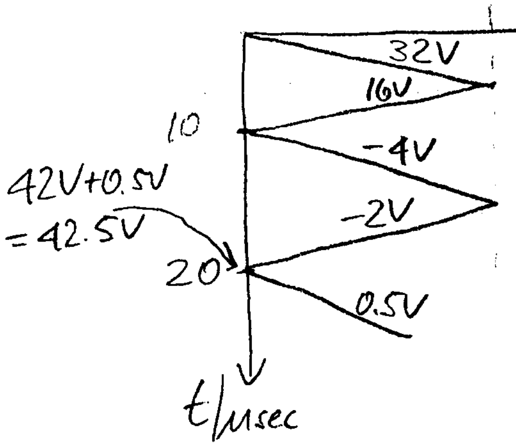
$$R_G = \frac{1+\rho_G}{1-\rho_G} Z_0 = \frac{3/4}{5/4} Z_0 = 0.6 Z_0 = \boxed{60\Omega}$$

$$R_T = \frac{1+\rho_T}{1-\rho_T} Z_0 = \frac{1.5}{0.5} Z_0 = 3 Z_0 = \boxed{300\Omega}$$

$$V_G = V_{0,1} \frac{R_G + Z_0}{Z_0} = 32V \frac{60+100}{100} = \boxed{51.2V}$$

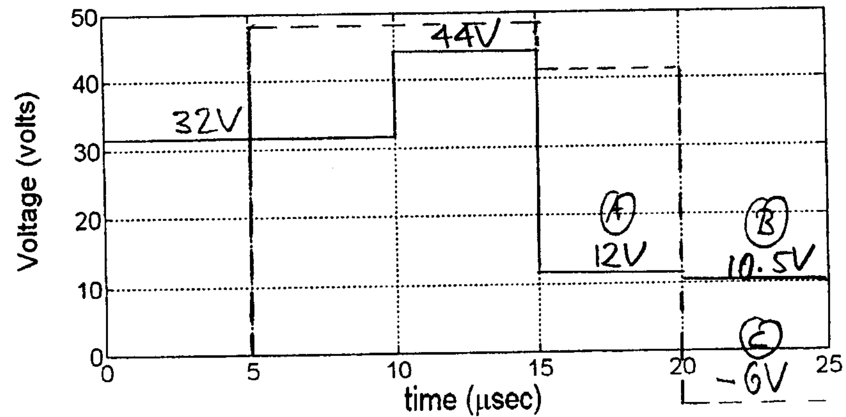
(f) Determine and sketch the voltage waveform $v_{in}(t, z=0)$ at the input of the line and $v_{out}(t, z=z_r)$ at the end of the line for $0 \le t \le 24\mu\text{sec}$ and specify the numerical voltage values. (Hint: first sketch a lattice diagram.)

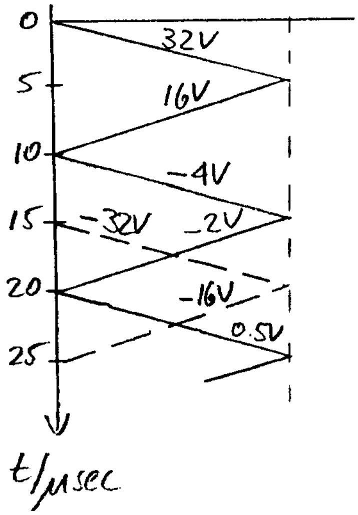




$\leftarrow$ 32V+16V-4V-2V = 42V

(g) Now, the step function generator is replaced by a **pulse generator**, which at time $t = 0$ produces a single rectangular impulse of duration $T = 15\mu\text{sec}$. Sketch the voltage $v_{in}(t, z=0)$ at the input of the line and $v_{out}(t, z=z_r)$ at the end of the line for $0 \le t \le 24\mu\text{sec}$ and specify the numerical voltage values.





Ⓐ 32V+16V-4V-32V = 12V

Ⓑ 32V+16V-4V-32V-2V+0.5V = 10.5V

Ⓒ 32V+16V-4V -2V-32V-16V = -6V

ECE 391 – TRANSMISSION LINES AND ELECTROMAGNETIC WAVES

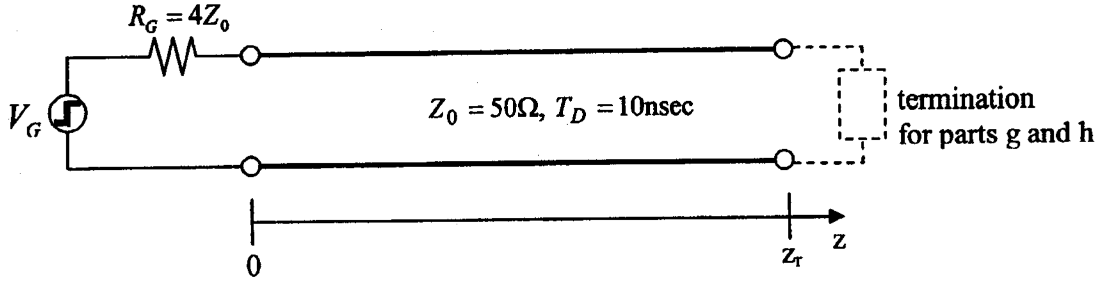
Spring Term 2005

Midterm I

Name: Solutions (Last name, first name) OSU-ID:________________

Exam is closed book, closed notes; ***one sheet*** *(2 pages) of formulas allowed; 50 minutes. Show all your work on the pages provided. No extra pages (use back if necessary). Read each question very carefully. Maximum number of points for each problem is given in parenthesis. Total: 40 points.*

1. (25 pts.) At time $t = 0$, a DC generator with internal resistance $R_G = 4Z_0$ and $V_G = 5\text{V}$ is connected to a lossless transmission line with characteristic impedance $Z_0 = 50\Omega$ and delay time $T_D = 10\text{nsec}$. The transmission line is open-circuited at the far end. The propagation velocity on the transmission line is 2×10^8 m/s.



(a) Determine the length of the line in meters.

$$z_T = V_p \cdot T_D = 2\times10^8 \tfrac{m}{s} \cdot 10\times10^{-9} s = \boxed{2m}$$

(b) Determine the voltage reflection coefficients at both ends.

$$\rho_G = \frac{R_G - Z_0}{R_G + Z_0} = \frac{4-1}{4+1} = \boxed{\frac{3}{5} = 0.6} \qquad \rho_T = \boxed{+1} \quad (\text{open circuit})$$

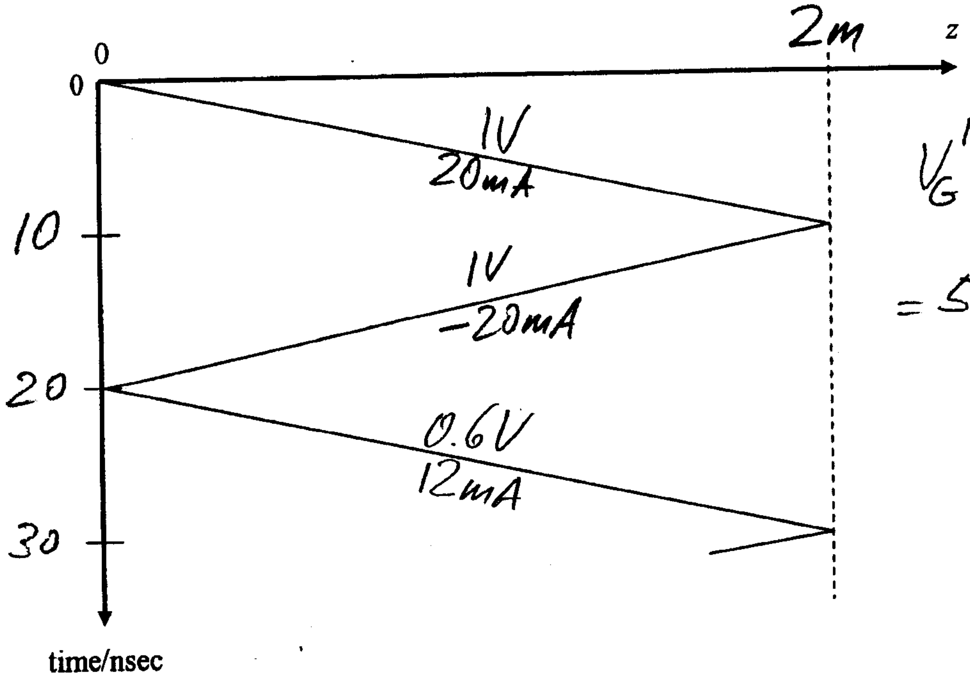
(c) Determine the distributed capacitance and inductance parameters of the transmission line.

from $Z_0 = \sqrt{\frac{L}{C}}$ and $V_p = \frac{1}{\sqrt{LC}}$ follows:

$$L = \frac{Z_0}{V_p} = \frac{50\Omega}{2\times10^8 \frac{m}{s}} = \boxed{250\,nH/m}$$

$$C = \frac{1}{Z_0 V_p} = \frac{1}{50\Omega\times2\times10^8\frac{m}{s}} = 10^{-10}\frac{F}{m} = \boxed{0.1\,nF/m}$$

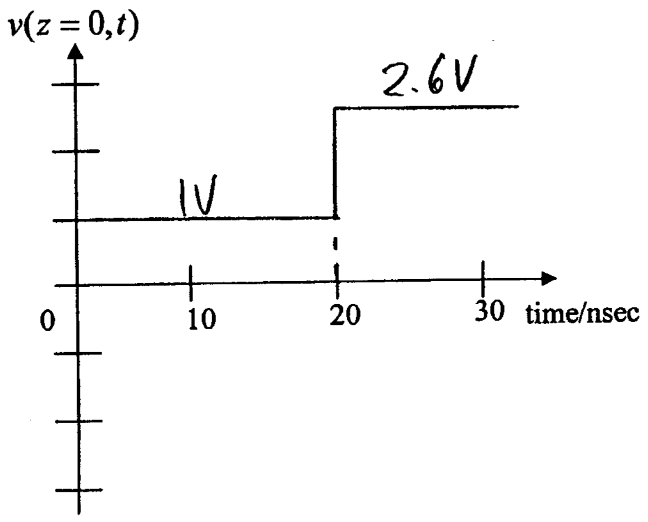
(d) Fill in the <u>numerical</u> **voltage** and **current** values for the first **three** wave components and add the time scale and length scale in the lattice diagram shown below.

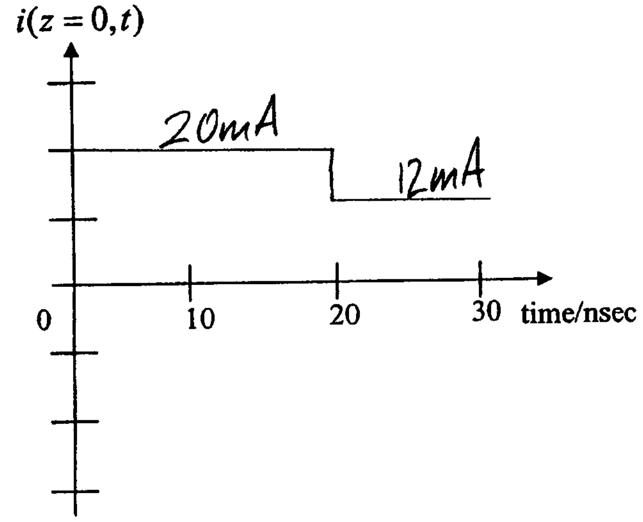


$$V_G' = V_G \frac{Z_0}{Z_0+R_G}$$

$$= 5V \frac{1}{1+4} = \boxed{1V}$$

(e) Sketch voltage $v(z=0,t)$ and current $i(z=0,t)$ at the <u>beginning of the line</u> $(z=0)$ for $0 \le t \le 30$ nsec. (Indicate the specific numerical values on the plots.)



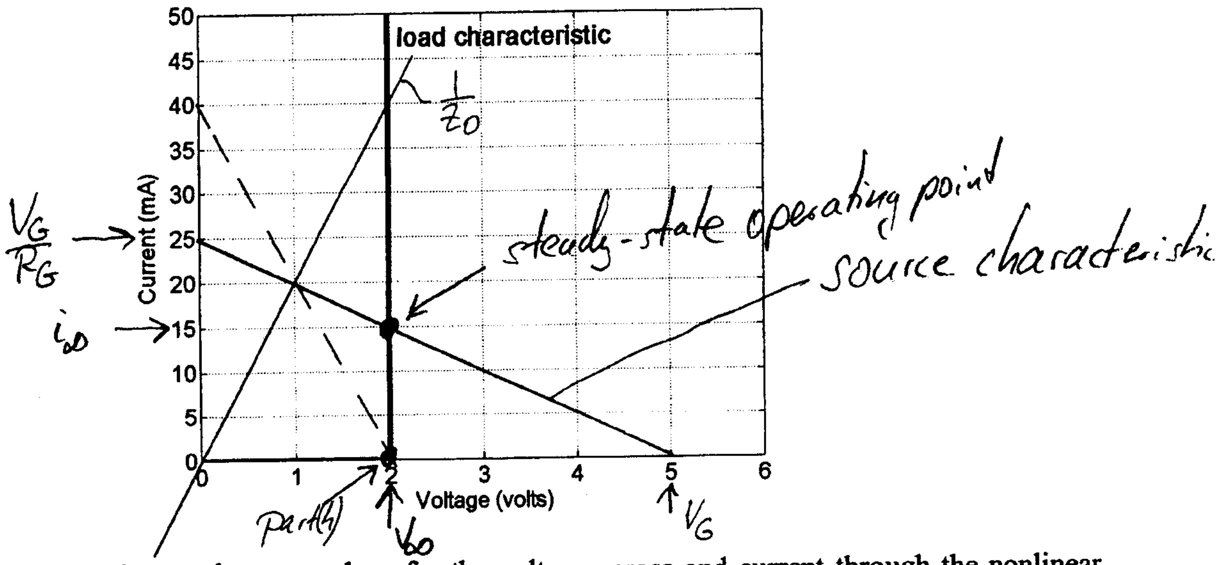


(f) Determine $v(z=0)$ and $i(z=0)$ at the input of the line for $t \to \infty$.

open circuit $\Rightarrow$ $V(t \to \infty) = \boxed{V_G = 5V}$

$i(t \to \infty) = \boxed{0}$

Now, the line is terminated with a nonlinear load whose I-V characteristic is shown below.



(g) Determine the steady-state values for the voltage across and current through the nonlinear load.

Source characteristic: $V_G = 5V$ $\quad i_{SC} = \frac{V_G}{R_G} = \frac{5V}{200\Omega} = 25mA$

$\Rightarrow \boxed{V_\infty = 2V \quad i_\infty = 15mA}$

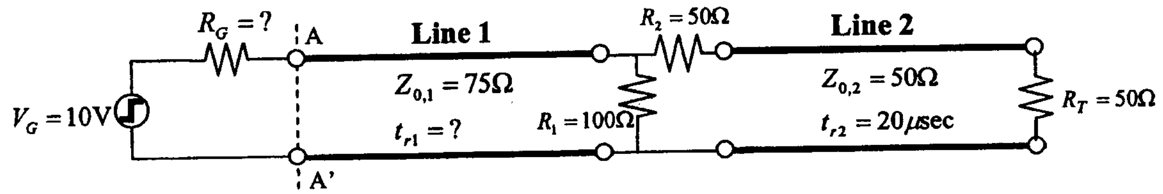
(h) Determine the voltage across and current through the nonlinear load right after the first outgoing wave reaches the load.

1) determine outgoing wave values → characteristic line through (0/0) with slope $1/Z_0$ → at 1V I = 20mA

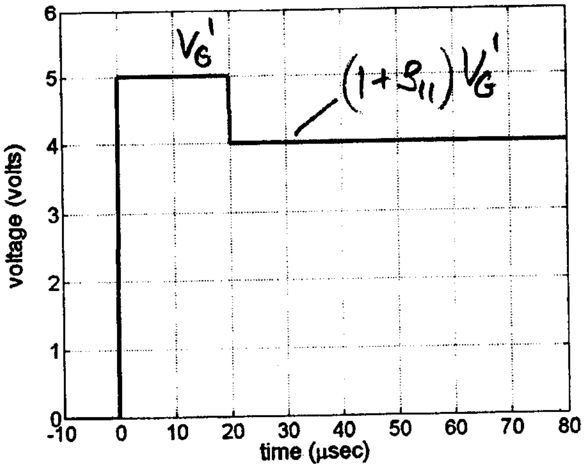
2) determine characteristic at the end of the line → goes through point (1V, 15mA) with slope $= -\frac{1}{Z_0}$

$\Rightarrow \boxed{V_{load} = 2V \quad i_{load} = 0}$ (from intersection with load characteristic)

2. (15 pts.) Two lossless transmission lines with different characteristic impedances are connected through a network consisting of two resistors, as shown below. The delay time of the first line is unknown, whereas the delay time of the second line is $20\mu sec$.



At time $t = 0$, a 10V battery is connected to the input of line 1 through an unknown generator resistance R_G. The voltage as a function of time shown below is observed on an oscilloscope connected at the input terminals of line 1 (at A-A').



(a) Determine the delay time t_{r1} of line 1.

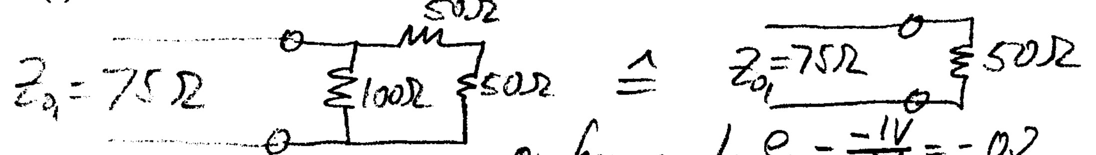
from graph: $2t_{r_1} = 20\mu sec \Rightarrow \boxed{t_{r_1} = 10\mu sec}$

(b) Determine the generator resistance R_G.

$V_G = 10V$ and $V_G' = 5V$ (from graph)

$\Rightarrow \boxed{R_G = Z_0 = 75\Omega}$

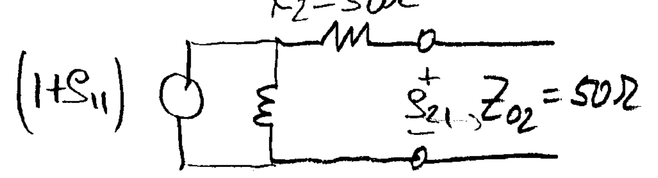
(c) Determine the reflection coefficient ρ_{11} at the junction between the two lines.



$$\rho_{11} = \frac{50-75}{50+75} = \boxed{-0.2}$$

Or from graph: $\rho_{11} = \frac{-1V}{5V} = -0.2$

(d) Determine the transmission coefficient ρ_{21} at the junction between the two lines.

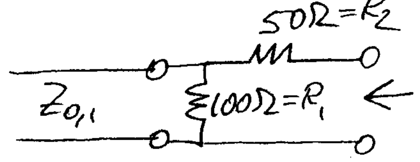


voltage divider

$$\rho_{21} = (1+\rho_{11})\frac{Z_{02}}{R_2+Z_{02}} = \frac{1}{2}(1+\rho_{11})$$

$$= \frac{1}{2}(1-0.2) = \boxed{0.4}$$

(e) Determine the reflection coefficient ρ_{22} and the transmission coefficient ρ_{12} at the junction between the two lines. (Recommendation: Skip to the next part if you are unsure and you are running out of time.)

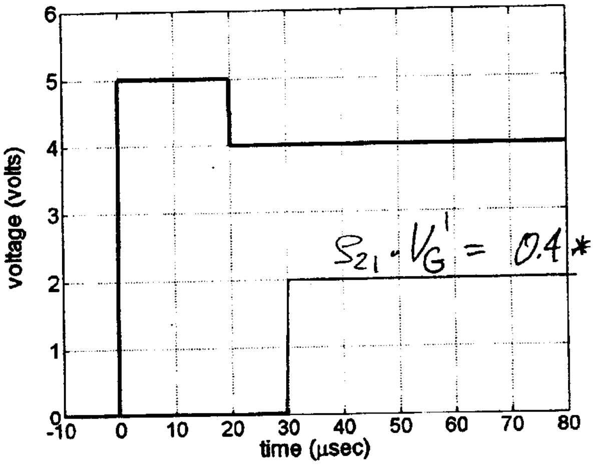


$$\leftarrow R_T = Z_{0,1} // 100\Omega + 50\Omega = 92.86\,\Omega$$

$$\rho_{22} = \frac{R_T - Z_{0,2}}{R_T + Z_{0,2}} = \boxed{0.3}$$

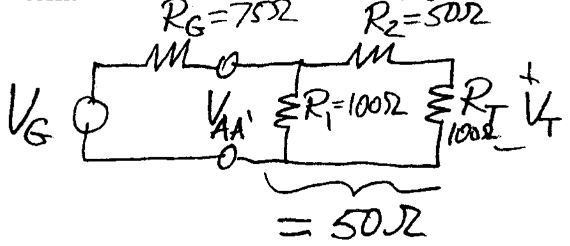
$$\rho_{12} = \frac{R_1 // Z_{0,1}}{R_1 // Z_{0,1} + R_2}(1+\rho_{22}) = \cdots = \boxed{0.6}$$

(f) Add the voltage waveform across the terminating resistor R_T for $-10 \le t \le 80\,\mu\text{sec}$ to the plot of the voltage at A-A' shown again below.



$$\rho_{21} \cdot V_G' = 0.4 * 5V = 2V$$

(g) Determine the steady-state voltages at the input of the first line (A-A') and across the termination of the second line (R_T) (i.e. for $t \to \infty$).



$$V_{AA'}(t\to\infty) = \frac{50\Omega}{50\Omega + 75\Omega} V_G = \frac{2}{5} V_G = \boxed{4V}$$

$$V_T(t\to\infty) = \frac{R_T}{R_2 + R_T} V_{AA'}(t\to\infty) = \frac{1}{2}\,4V = \boxed{2V}$$

↑ compare with (f)!

ECE 391 – TRANSMISSION LINES AND ELECTROMAGNETIC WAVES

Spring Term 2005

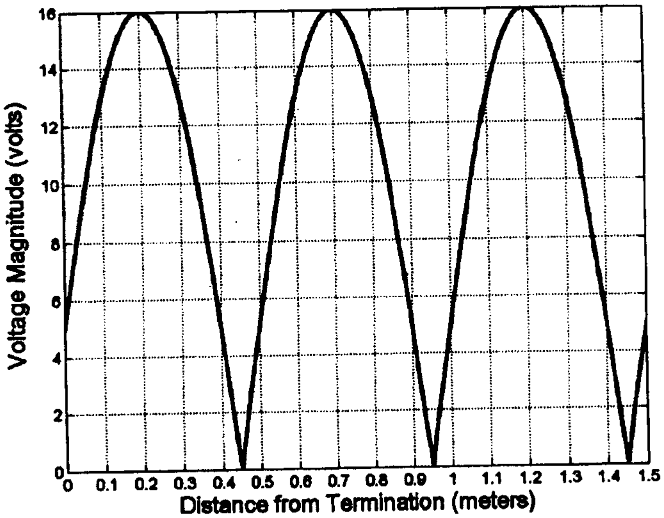
Midterm II

Name: Solutions Student ID ____________________

(Last name, first name)

Exam is closed book, closed notes; **one** sheet (2 pages) of formulas allowed; 50 minutes. Show all your work on the pages provided. No extra pages (use back if necessary). Read each question very carefully. Maximum number of points for each problem is given in parenthesis. Total: 40 points.

1. (12 pts.) A 50Ω coaxial transmission line (filled with air) is terminated in an **unknown** load impedance Z_T. The voltage standing-wave on the transmission line measured in meters from the termination is shown below.



(a) What is the standing-wave ratio on the line terminated in the unknown load impedance Z_T?

$$SWR = \frac{V_{max}}{V_{min}} = \infty$$

(b) Determine the wavelength on the line.

distance between adjacent maxima or minima $= \frac{\lambda}{2}$

here. $\frac{\lambda}{2} = (0.7-0.2)m = 0.5m$

$$\boxed{\lambda = 1m}$$

(c) Determine the voltage amplitude $|V_m^+|$ of the incident wave.

$$|K| = 1 \Rightarrow V_m^+ = \frac{1}{2} V_{max} = \boxed{8V}$$

$$\text{Or } V_m^+ = \frac{1}{2}(V_{max} + V_{min})$$

(d) Determine the reflection coefficient at the termination (**magnitude** and **phase**).

$$K = |K| e^{j\theta_K}$$

$$SWR = \infty \Rightarrow \boxed{|K| = 1}$$

$$\frac{\theta_K}{2} = \text{shift of maximum voltage} = \frac{0.2m}{\lambda} 360^0$$

$$= 72^0$$

$$\Rightarrow \theta_K = 2 * 72^0 = \boxed{144^0}$$

Or: from Smith chart

1) V_{max} at $\phi = 0$ ($\hat{=}$ 0.25 on "toward load" scale)
2) go 0.2λ toward load $\rightarrow$ 0.45
3) read off $\theta_K = 144^0$

(e) Determine the load impedance Z_T in ohms. (An <u>optional</u> Smith Chart is attached at the end.)

$$Z_T = Z_0 \frac{1+K}{1-K} = \boxed{j\ 16.25\Omega}$$

or from Smith chart

$$z_T = j\,0.325 \Rightarrow Z_T = z_T Z_0 = j16.25\Omega$$

2. (20 pts.) A lossless transmission line of characteristic impedance $Z_0 = 100\Omega$ is terminated in a complex impedance $Z_T = R_T + jX_T$. The normalized load impedance $z_T = Z_T / Z_0$ is specified on the attached Smith Chart. Use the attached Smith Chart to solve all of the following problems. **Show all your work on the Smith Chart to get points!**

(a) Determine the normalized load impedance $z_T = Z_T / Z_0$ and give the terminating impedance $Z_T = R_T + jX_T$ in ohms.

$z_T = 0.5 - j0.6 \Rightarrow Z_T = z_T \cdot Z_0 = (50 - j60)\Omega$

(b) Determine the corresponding normalized load admittance and load admittance in siemens (clearly show your work on the Smith chart!).

from Smith chart

$y_T = 0.825 + j0.975 \Rightarrow Y_T = \frac{y_T}{Z_0} = (0.00825 + j0.00975)\,S$

just for comparison

(with calculator: $Y_T = 0.0082 + j0.0098\,S$)

(c) Determine the reflection coefficient (magnitude and phase in degrees) at the termination.

from Smith chart:

$\boxed{|K| = 0.48}$ $\boxed{\theta_K = -108^\circ}$

note: if you determine θ_K from Y_T, you have to include the change from K to $-K$ i.e. add 180°

(d) Determine the voltage standing-wave ratio on the line.

from Smith chart:

$\boxed{SWR = 2.8}$

(e) Determine the distance $d_{i,\min}/\lambda$ to the nearest **current minimum** from the termination.

current minimum occurs at voltage maximum

$\Rightarrow$ voltage maximum at $\phi = 0$

$\Rightarrow$ go from 0.4 to 0.5 to 0.25

$\Rightarrow \frac{d_{i,\min}}{\lambda} = 0.5 - 0.4 + 0.25 = \boxed{0.35}$

(f) Determine the input impedance $Z_{in}(d_1) = R_{in} + jX_{in}$ in ohms at distance $d_1 = 1.228\,\lambda$ from the termination.

go from 0.4 to $0.4 + 1.228 = 1.628 \triangleq 0.128$
on "towards generator" scale

read off: $z_{in} \cong 0.63 + j0.82 \Rightarrow Z_{in} = z_{in} \cdot Z_0 = (63 + j82)\Omega$

(g) A resistor $R_s = 70\Omega$ is inserted **in series** at distance $d_1 = 1.228\lambda$ from the termination. Determine the new standing-wave ratio on the line for $d > d_1$ (i.e., on the line segment closer to the generator).

add R_s in series:
1) normalize $r_s = \frac{R_s}{Z_0} = 0.7$
2) add $r_s = 0.7$ to $r_{in} = 0.63$
3) new $r = r_s + r_{in} = 1.33$
4) x_{in} remains unchanged
5) draw $|K| =$ const circle through new impedance
6) read off new SWR

$\Rightarrow \boxed{SWR = 2.1}$

(h) Now, assume that the operating frequency is increased by 20% and the terminating impedance remains unchanged. Determine the new input impedance at distance d_1 from the termination **without** the series connected resistor. Hint: the <u>physical</u> distance d remains unchanged.

- increase in f by factor 1.2 means
decrease in λ by factor $\frac{1}{1.2}$ and
increase in $\frac{d}{\lambda}$ by factor 1.2

new $\frac{d_1}{\lambda} = 1.228 * 1.2 = 1.4736 \approx 1.474$

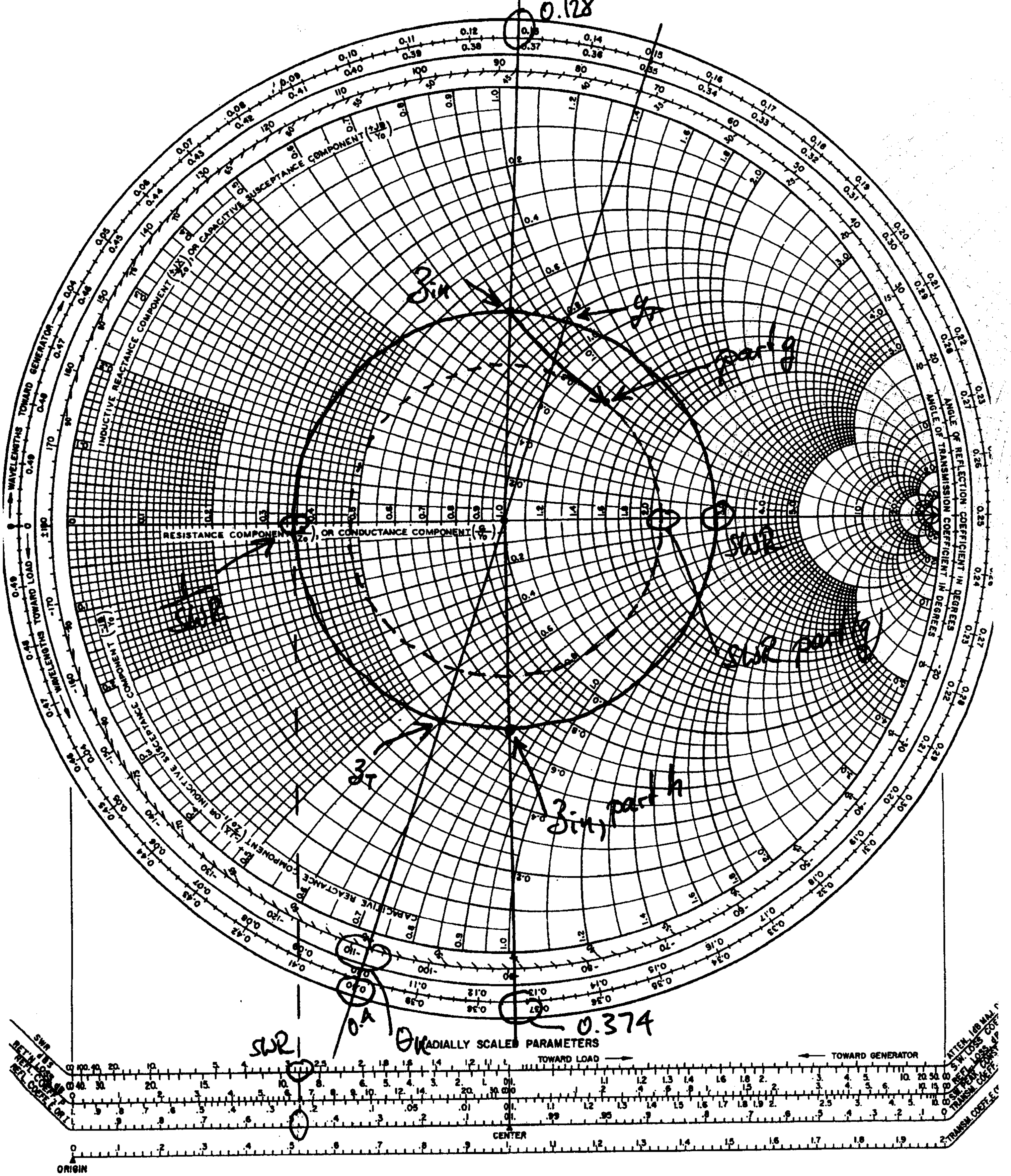
on "toward generator scale" go from 0.4 to
$0.4 + 1.474 = 1.874 \triangleq 0.374$

$\Rightarrow$ read off $\boxed{z_{in} = 0.63 - j0.78}$

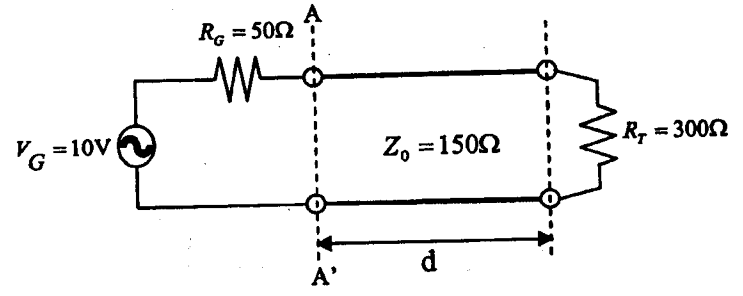
$(Z_{in} = 63 - j78\,\Omega)$

Smith chart for problem 2

IMPEDANCE OR ADMITTANCE COORDINATES



3. (8 pts.) A source with a 50Ω generator resistance is connected to a resistive load $R_T = 300\Omega$ through a section of lossless transmission line of length $d = 2$ meters and characteristic impedance $Z_0 = 150\Omega$. The propagation velocity on the line is $v_p = 2\times10^8$ m/s.



(a) What is the voltage standing-wave ratio on the line?

$$SWR = \frac{R_T}{Z_o} = 2 \quad (\text{since } R_T > Z_o)$$

(b) Determine the **lowest** frequency $f = f_0 > 0$ at which the generator sees a purely resistive load impedance at terminals A-A'.

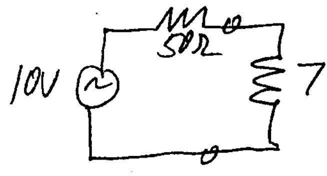
for $d = \frac{\lambda}{4}$ line transforms from real to real impedance

$$\Rightarrow \lambda = 4d = \boxed{8m}$$

$$f_o = \frac{v_p}{\lambda} = \frac{2\times10^8 \frac{m}{s}}{8m} = \boxed{25\ MHz}$$

(c) Determine the average power dissipated in the terminating resistance R_T at frequency $f = f_0$ determined in part b.

$$Z_{in} = \frac{Z_o^2}{R_T} = \frac{(150\Omega)^2}{300\Omega} = 75\Omega$$



$$V_{in} = 10V \frac{75}{50+75} = 6V$$

$$P_{ave} = \frac{1}{2}\frac{V_{in}^2}{R_{in}} = \frac{1}{2}\frac{(6V)^2}{75\Omega} = \boxed{0.24W}$$

(d) Determine the average power dissipated in the terminating resistance R_T at frequency $f = 2f_0$.

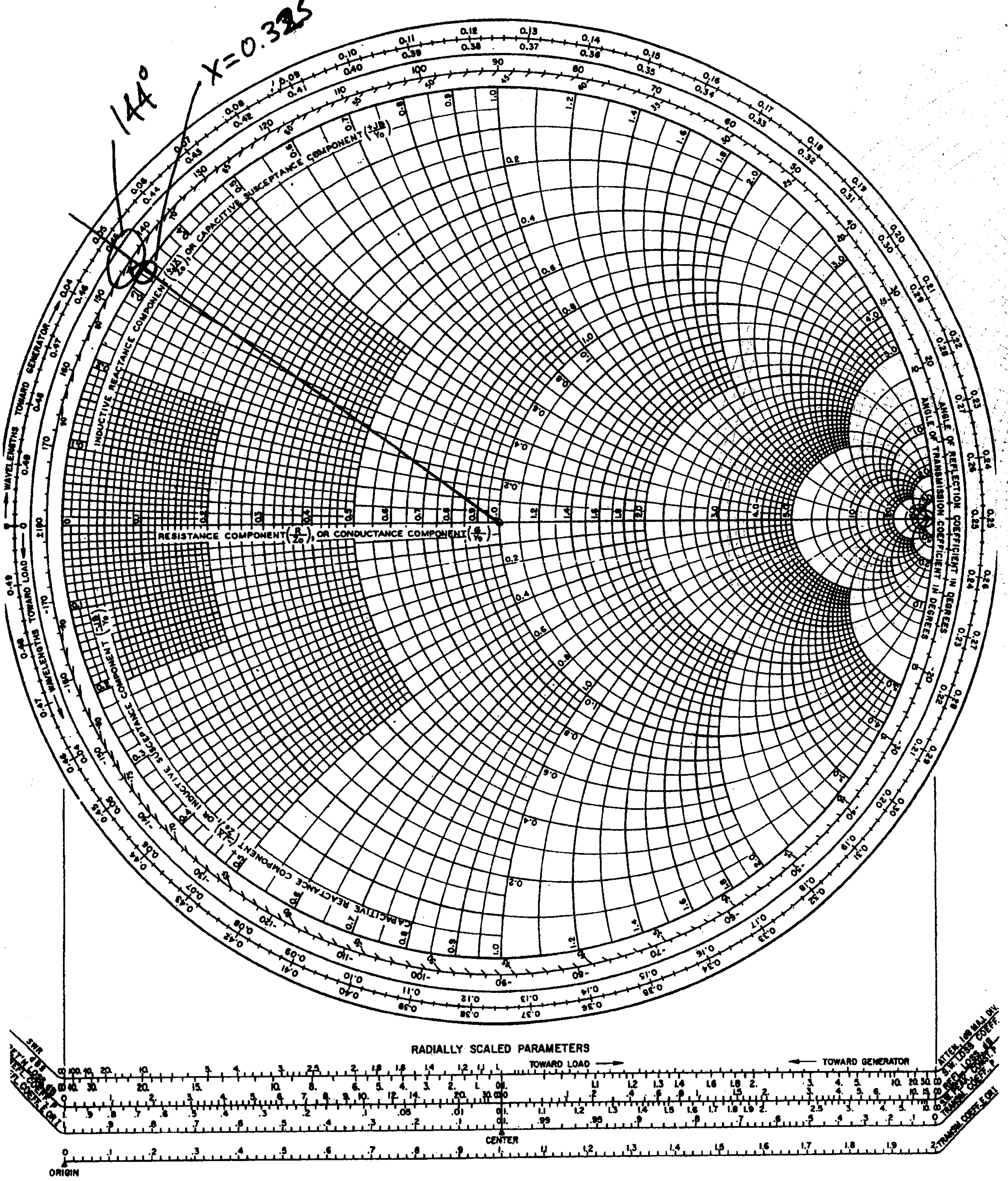
$$\text{at } f = 2f_o \quad d = \frac{\lambda}{2}$$

$$\Rightarrow R_{in} = 300\Omega \quad V_{in} = 10V\frac{300}{50+300} = 8.57V$$

$$P_{ave} = \frac{1}{2}\frac{|V_{in}|^2}{R_{in}} = \boxed{0.122\ W}$$

Optional Smith chart

IMPEDANCE OR ADMITTANCE COORDINATES



ECE 391 – TRANSMISSION LINES AND ELECTROMAGNETIC WAVES

Winter Term 2004

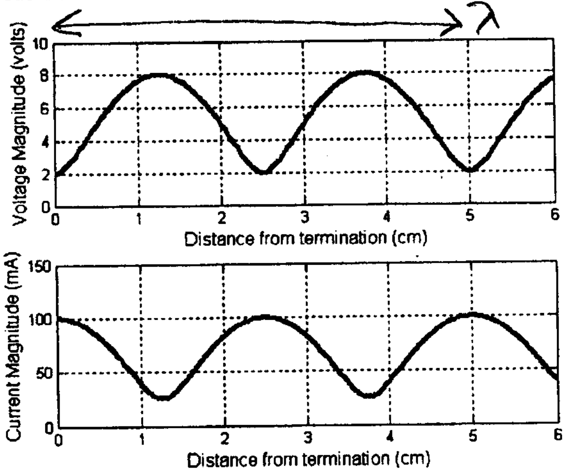
Midterm II

Name: Solutions Student ID: xxx-xx-__________

(Last name, first name)

Exam is closed book, closed notes; **one** sheet (2 pages) of formulas allowed; 50 minutes. Show all your work on the pages provided. No extra pages (use back if necessary). Read each question very carefully. Maximum number of points for each problem is given in parenthesis. Total: 40 points.

1. (14 pts.) A transmission line of **unknown** characteristic impedance Z_0 is terminated in an **unknown** load impedance Z_T. The voltage and current standing-wave patterns along the transmission line as function of distance from the termination are shown below.



Answer the following questions. An optional Smith chart is also attached.

(a) What is the standing-wave ratio on the line terminated in the unknown load impedance Z_T?

$$SWR = \frac{8V}{2V} = \boxed{4}$$

(b) Determine the wavelength on the line.

$$\boxed{\lambda = 5cm}$$

(c) Determine the operating frequency f_0 in Hz if the velocity factor of the transmission line is 50%.

$$V_p = 0.5c = 1.5 \times 10^8 \frac{m}{s}$$

$$f_0 = \frac{V_p}{\lambda} = \frac{1.5 \times 10^8 \frac{m}{s}}{0.05m} = \boxed{3\ GHz}$$

(d) Determine the characteristic impedance of the transmission line.

$$Z_0 = \frac{V_{max}}{I_{max}} = \frac{8V}{100mA} = \boxed{80\Omega}$$

(e) Determine the magnitude and phase (in degrees) of the reflection coefficient at the termination.

$$|K| = \frac{SWR-1}{SWR+1} = \frac{3}{5} = \boxed{0.6}$$

minimum at termination $\Rightarrow \boxed{\theta_K = 180^\circ}$

or: distance to first maximum $= \frac{\theta_K}{2} \triangleq \frac{1.25cm}{5cm}\lambda$

$$= \frac{1}{4}\lambda \triangleq 90^\circ \Rightarrow \theta_K = 2 \times 90^\circ = 180^\circ$$

(f) Determine the load impedance in ohms.

Z_T is purely real

$$K = |K| e^{j180^\circ} = -|K| = -0.6 \Rightarrow R_T = Z_0 \frac{1+K}{1-K} = 20\Omega$$

$$\text{or: } Z_T = Z_{min} = \frac{Z_0}{SWR} = \frac{80\Omega}{4} = \underline{\underline{20\Omega}}$$

(g) Determine how the voltage standing-wave pattern would change if the load impedance were removed (i.e., for an open-circuited line).

for an open circuit: $V = V_{max,oc}$ at the termination

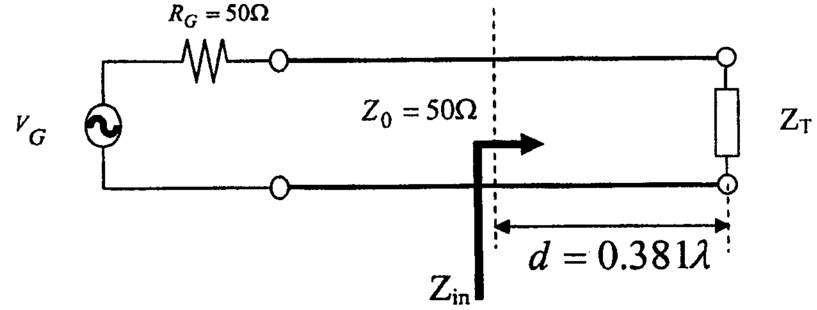
With $V_{max,oc} = 2V^+$

$$V^+ = \frac{1}{2}(8V + 2V) = 5V$$

$$\therefore V_{max,oc} = 2 \times 5V = \boxed{10V}$$

$$V_{min} = \boxed{0V}$$

2. (17 pts.) A lossless transmission line of characteristic impedance $Z_0 = 50\Omega$ is terminated in the complex impedance $Z_T = R_T + jX_T$. The normalized input impedance $z_{in} = Z_{in} / Z_0$ at distance $d = 0.381\lambda$ from the termination is specified on the attached Smith Chart. Use the attached Smith Chart to solve all of the following problems. **Show all your work on the Smith Chart!**



(a) From the Smith Chart, find the normalized input impedance $z_{in} = r_{in} + jx_{in}$ and give the input ~~terminating~~ impedance $Z_{in} = R_{in} + jX_{in}$ in ohms.

$$z_{in} = \boxed{0.5 - j0.8} \Rightarrow Z_{in} = z_{in} Z_0 = \boxed{(25 - j40)\,\Omega}$$

(b) Determine the voltage standing-wave ratio on the line using the Smith chart. (Clearly indicate your solution approach on the Smith chart.)

from Smith chart: $SWR \approx 3.5$ (intersection with $x=0$ on right half)

or $\frac{1}{SWR} \approx 0.29$

$\Rightarrow SWR = 3.45$

(c) Determine the reflection coefficient (magnitude and phase in degrees) at distance d from the termination using the Smith Chart.

$$K_{in}(z'=d) = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \boxed{0.55\, e^{-j94^0}}$$ from Smith chart

Note: For comparison, exact calculation gives

$$K_{in} = 0.5549\, e^{-j93.933^0}$$

(d) Using the Smith Chart and <u>starting at the location where the input impedance is known</u> (distance d from the termination), determine the distance to the nearest voltage <u>minimum</u> $d_{v,\min} / \lambda$ in the direction to the load.

To get to the nearest voltage minimum going toward the load, rotate in counter-clockwise direction until the phase of the reflection coefficient is 180°

$$\Rightarrow \text{go from } 0.119 \text{ to } 0.5 \Rightarrow \frac{d_{v,min}}{\lambda} = 0.5 - 0.119 = \boxed{0.381}$$

(e) Determine the normalized load impedance $z_T = r_T + jx_T$ at the termination and give the terminating impedance $Z_T = R_T + jX_T$ in ohms.

for $d = 0.381\lambda$, $Z_T = Z_{min}$ (from previous problem)

$$\Rightarrow z_T = z_{min} = \frac{1}{SWR} \hat{=} 0.29$$

$$\text{and } Z_T = z_T Z_0 \hat{=} \boxed{14.5\Omega}$$

(f) Using the Smith chart, determine the load impedance if a capacitance with normalized impedance $z_c = -jx_c =$ [illegible] $-j0.5$ is connected in parallel to impedance $Z_T = R_T + jX_T$. (Show all steps.)

parallel connection $\Rightarrow$ convert to admittances

$$y_T = \frac{1}{z_T} \overset{here}{\hat{=}} SWR \hat{=} 3.5$$

$$y_c = \frac{1}{z_c} = j2$$

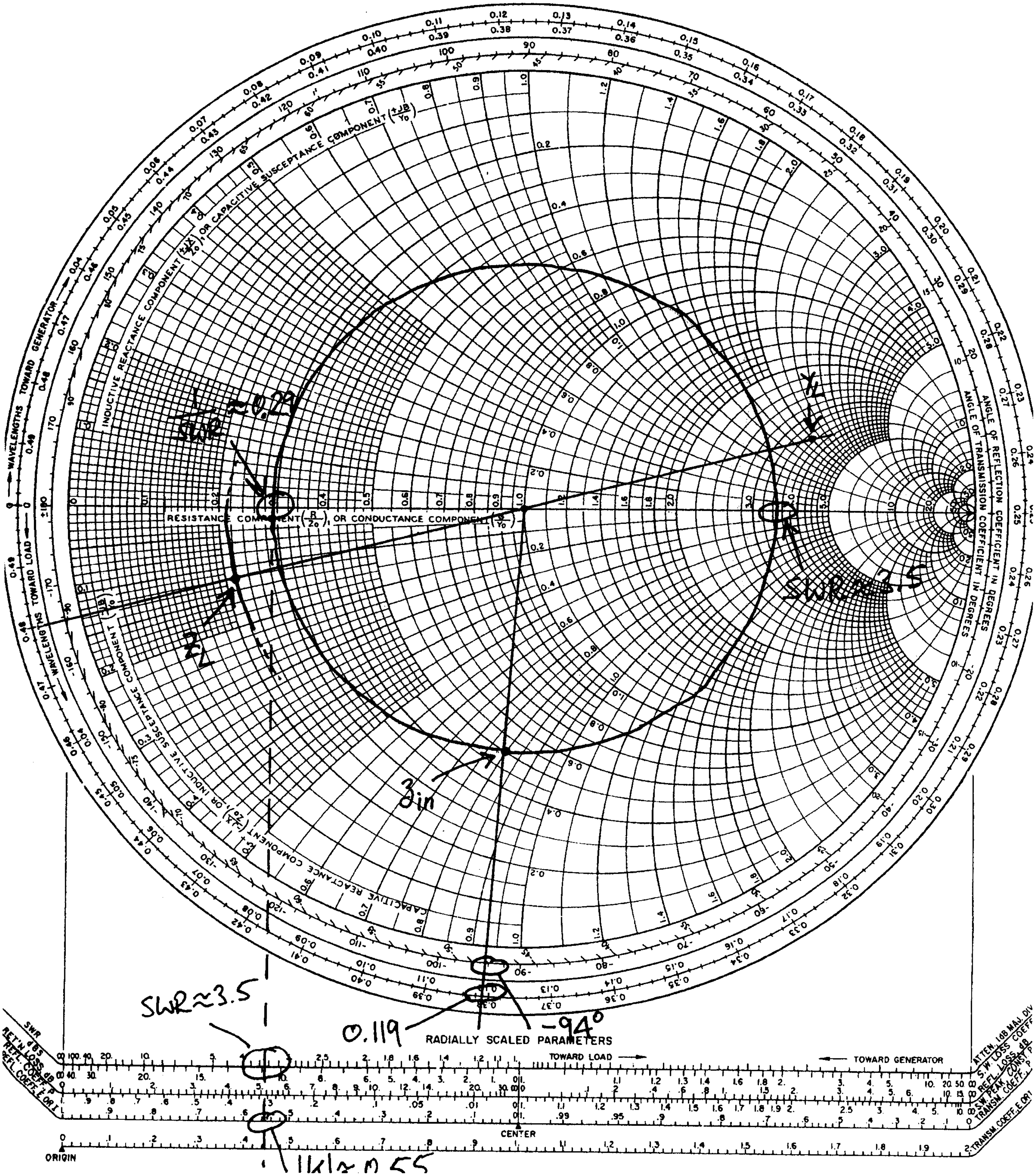
$$Y_L = y_c + y_T = 3.5 + j2$$

$$z_L = \frac{1}{y_L} \hat{=} \boxed{2.2 - j0.12}$$ (from Smith chart!)

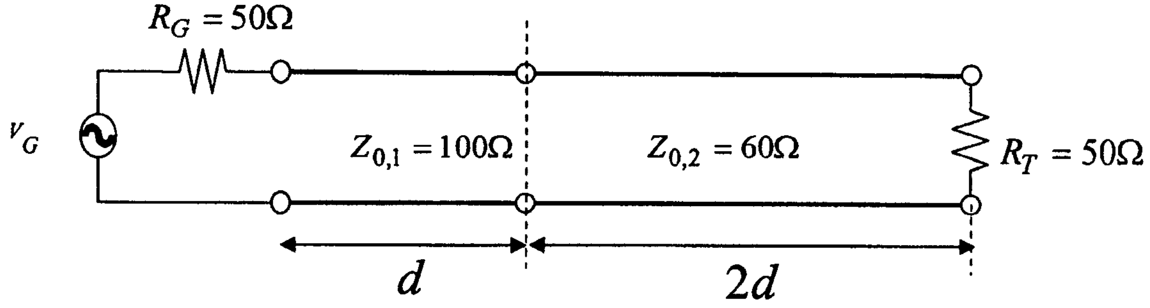
$$Z_L = z_L Z_0 \hat{=} \boxed{(110 - j6)\Omega}$$

NAME	TITLE'	DWG. NO.
SMITH CHART FORM 82-BSPR(9-66)	KAY ELECTRIC COMPANY, PINE BROOK, N.J., © 1966. PRINTED IN U.S.A.	DATE

IMPEDANCE OR ADMITTANCE COORDINATES



3. (9 pts.) A resistive load $R_T = 50\Omega$ is connected through two cascaded transmission line sections to a generator with source resistance $R_G = 50\Omega$ as shown below. At frequency f_0 the line length d corresponds to a quarter wavelength ($d = \lambda/4$).



(a) What is the standing-wave ratio on the left transmission-line segment with characteristic impedance $Z_{0,1} = 100\Omega$ at operating frequency $f = f_0$ ($d = \lambda/4$)?

for $f = f_0 \Rightarrow d = \lambda/4$ and $2d = \lambda/2$

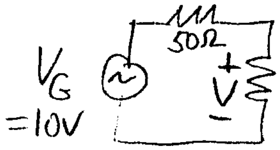
$\Rightarrow$ load impedance seen by left line is 50Ω

$$\Rightarrow SWR = \frac{100\Omega}{50\Omega} = \boxed{2} \quad \left(= \frac{1}{S_{min}}\right)$$

(b) What is the average power dissipated in load resistance $R_T = 50\Omega$ if the source has a peak voltage of 10V and is operating at frequency $f = f_0$?

input impedance to left line is obtained by $\lambda/4$ transformation

$$Z_{in} = \frac{Z_{0,1}^2}{50\Omega} = \frac{(100\Omega)^2}{50\Omega} = 200\Omega$$



$$\Rightarrow P_{ave} = \frac{1}{2}\frac{V^2}{200\Omega} = \frac{1}{2}\left(V_G \frac{200}{250}\right)^2 \frac{1}{200\Omega}$$
$$= \frac{1}{2}\frac{(8V)^2}{200\Omega} = \boxed{0.16W}$$

(c) What is the average power dissipated in load resistance $R_T = 50\Omega$ if the operating frequency is doubled ($f = 2f_0$)?

for $f = 2f_0$, $d = \frac{\lambda}{2}$ and $2d = \lambda$

$\Rightarrow Z_{in} = 50\Omega$ and

$$P_{ave} = \frac{1}{2}\frac{(5V)^2}{50\Omega} = \boxed{0.25W}$$

ECE 391 – TRANSMISSION LINES AND ELECTROMAGNETIC WAVES

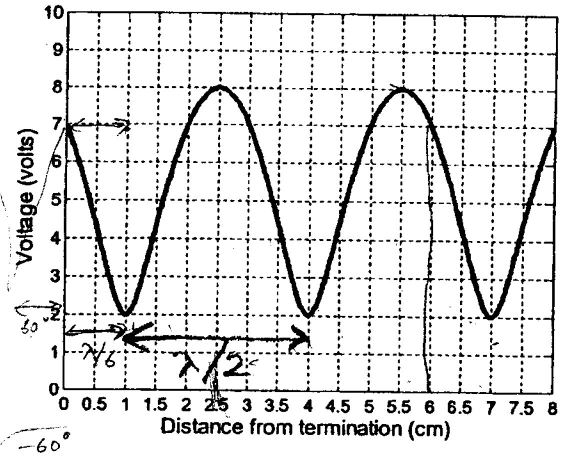
Spring Term 2003

Midterm II

Name: Solutions (Last name, first name) SSN: xxx-xx-__________

Exam is closed book, closed notes; **one** sheet (2 pages) of formulas allowed; 50 minutes. Show all your work on the pages provided. No extra pages (use back if necessary). Read each question very carefully. Maximum number of points for each problem is given in parenthesis. Total: 40 points.

1. (13 pts.) A 50Ω transmission line is terminated in an **unknown** load impedance Z_T. The voltage standing-wave on the transmission line measured in electrical distance from the termination is shown below. Answer the following questions. An optional Smith chart is also attached.



$\lambda = 360°$

$\frac{\lambda}{6} = 60°$

$-60°$

$\left(\frac{2.5}{6}\right)(360°)$

V_{max}

$\frac{\theta_\Gamma}{2} = 150°$

$\theta_\Gamma - 360°$ or $-60°$

(a) Determine the wavelength on the line.

$\lambda/2 = 4cm - 1cm = 3cm$

$\Rightarrow \boxed{\lambda = 6cm}$

(b) What is the standing-wave ratio on the line terminated in the unknown load impedance Z_T?

$SWR = \frac{V_{max}}{V_{min}} = \frac{8V}{2V} = \boxed{4}$

$V_{min} = -60°$
$\phi_m = \pm 180° = \frac{\theta_K}{2}$

(c) Determine the magnitude and phase (in degrees) of the reflection coefficient at the termination.

$$|K_T| = \frac{SWR-1}{SWR+1} = \frac{3}{5} = \boxed{0.6} \qquad \frac{\partial K}{2} =$$

phase calculation

distance to 1st voltage minimum $= l_{min} = \frac{\lambda}{6} \hat{=} 60°$

$$\phi_{V_{min}} = \pm 180° = \theta_K - 2 \times 60° \Rightarrow \theta_K = \pm 180° + 120° = \boxed{\begin{cases} 300° \\ -60° \end{cases}} \text{ sam}$$

or from Smith chart: draw SWR = 4 circle
from V_{min} location, go $\frac{1}{6}\lambda = 0.1667\lambda$ towards load $\rightarrow \boxed{\phi = -60°}$

$$K = \frac{Z_T - Z_0}{Z_T + Z_0}$$

(d) Determine the load impedance in ohms.

$$Z_T = Z_0 \frac{1+K_T}{1-K_T} \hat{=} (42.1 - j68.4)\,\Omega \leftarrow Z_T = 50\left(\frac{1+0.6e^{j60°}}{1-0.6e^{j60°}}\right)$$

arbitrary complex termination

or, from Smith chart: $z_T \hat{=} 0.84 - j1.37$

$$\Rightarrow Z_T = Z_0 z_T \hat{=} \boxed{(42 - j68.5)\,\Omega}$$

(e) Determine the voltage amplitude V_m^+ of the outgoing wave.

$$V_m^+(1+|K|) = 8V = V_{max}$$

$$\Rightarrow V_m^+ = \frac{8V}{1+|K|} = \frac{8V}{1.6} = \boxed{5\,V}$$

Or: $V_{max} + V_{min} = V_m^+(1+|K|) + V_m^+(1-|K|) = 2V_m^+$

$$\Rightarrow V_m^+ = \frac{V_{max} + V_{min}}{2} = \frac{8V + 2V}{2} = \underline{\underline{5}}$$

(f) What is the average power (in watts) dissipated in the terminating impedance Z_T?

$$P_{ave} = \frac{1}{2}\frac{|V_m^+|^2}{Z_0}(1 - |K|^2)$$

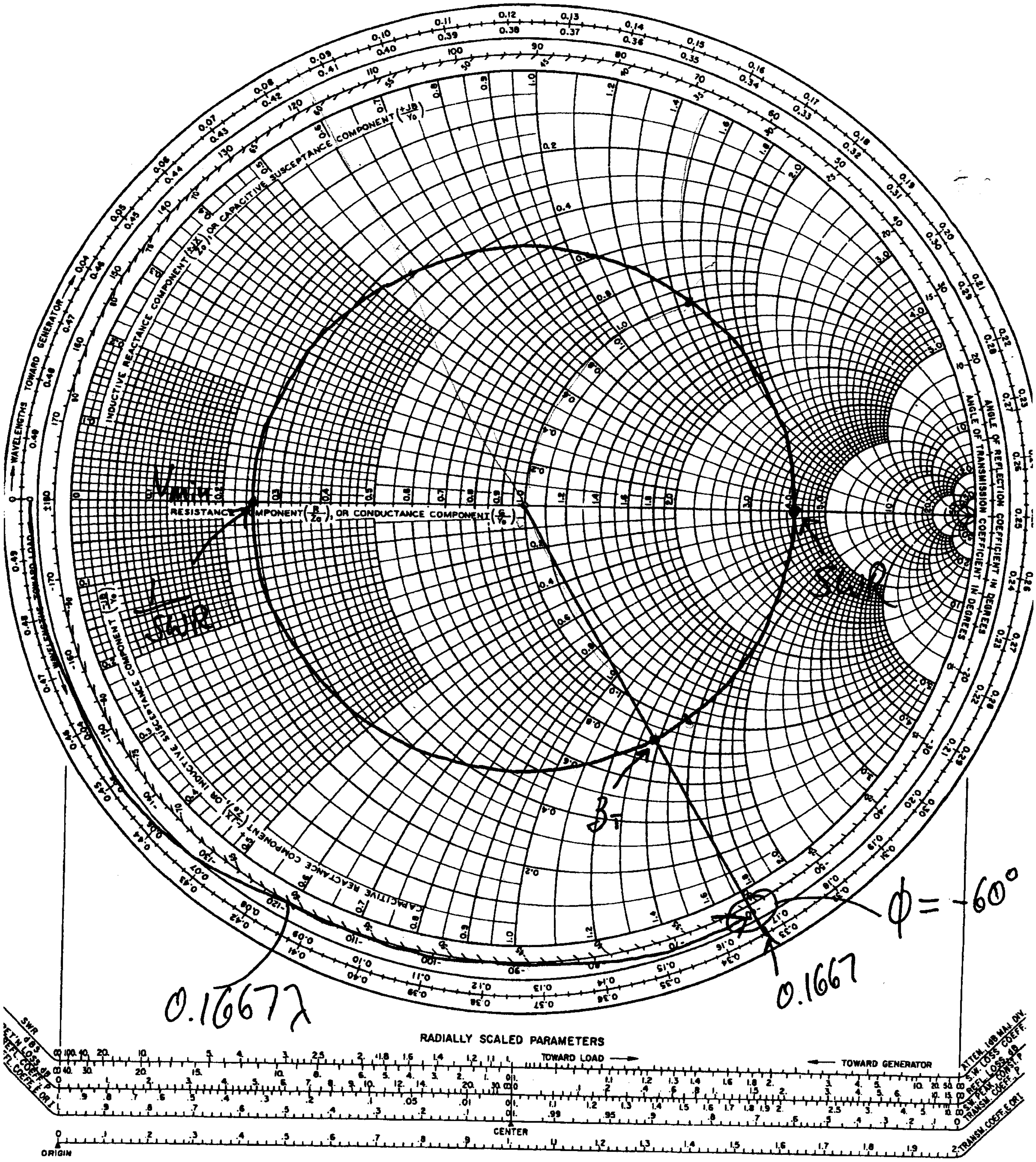
$$= \frac{1}{2}\frac{(5V)^2}{50\Omega}(1 - 0.36) = \boxed{160\,mW}$$

$$\frac{60 \cdot 2\pi}{360}$$

$$= \frac{2\pi}{6} \quad \frac{\pi}{3}$$

Optional Smith chart for problem 1

IMPEDANCE OR ADMITTANCE COORDINATES



2. (17 pts.) A lossless transmission line of characteristic impedance $Z_0 = 100\Omega$ is terminated in the complex impedance $Z_T = R_T + jX_T$. The normalized terminating impedance $z_T = Z_T / Z_0$ is specified on the attached Smith Chart. Use the attached Smith Chart to solve all of the following problems. **Show all your work on the Smith Chart!**

(a) From the Smith Chart, find the normalized terminating impedance $z_T = r + jx$ and give the terminating impedance $Z_T = R_T + jX_T$ in ohms.

$$z_T = \boxed{0.5 - j0.7} \Rightarrow Z_T = Z_0 z_T = \boxed{(50 - j70)\Omega}$$

(b) Determine the voltage standing-wave ratio on the line using the Smith chart. (Clearly indicate your solution approach on the Smith chart.)

$$SWR \cong 3.2$$

(c) Determine the reflection coefficient (magnitude and phase in degrees) at the termination using the Smith Chart.

$$|K| \approx \boxed{0.52}$$

$$\theta_K \approx \boxed{-101^0}$$

(d) Determine the distance to the nearest voltage <u>minimum</u> $d_{v,\min} / \lambda$ from the termination using the Smith Chart.

$$\frac{d_{v,min}}{\lambda} \doteq 0.5 - 0.39 = \boxed{0.11}$$

Now, the complex termination is to be matched to the transmission line using a variable series-connected capacitor.

(e) Determine the position d/λ at which the capacitor is to be connected <u>in series</u> to the transmission line (using the Smith chart).

go on SWR = 3.2 ($|\Gamma| = 0.52$) circle in clockwise direction until you intersect with r = 1 circle. Look for inductive solution

$$\frac{d}{\lambda} = (0.5-0.39) + \underset{0.168}{\cancel{0.139}} = \boxed{0.278} \quad \cancel{0.249}$$

(f) Determine the capacitance value C to achieve a match at the design frequency $f = 100\,\text{MHz}$.

$$z_{in} \triangleq 1 + j1.22$$

$$Z_{in} = (100 + j\,122)\,\Omega$$

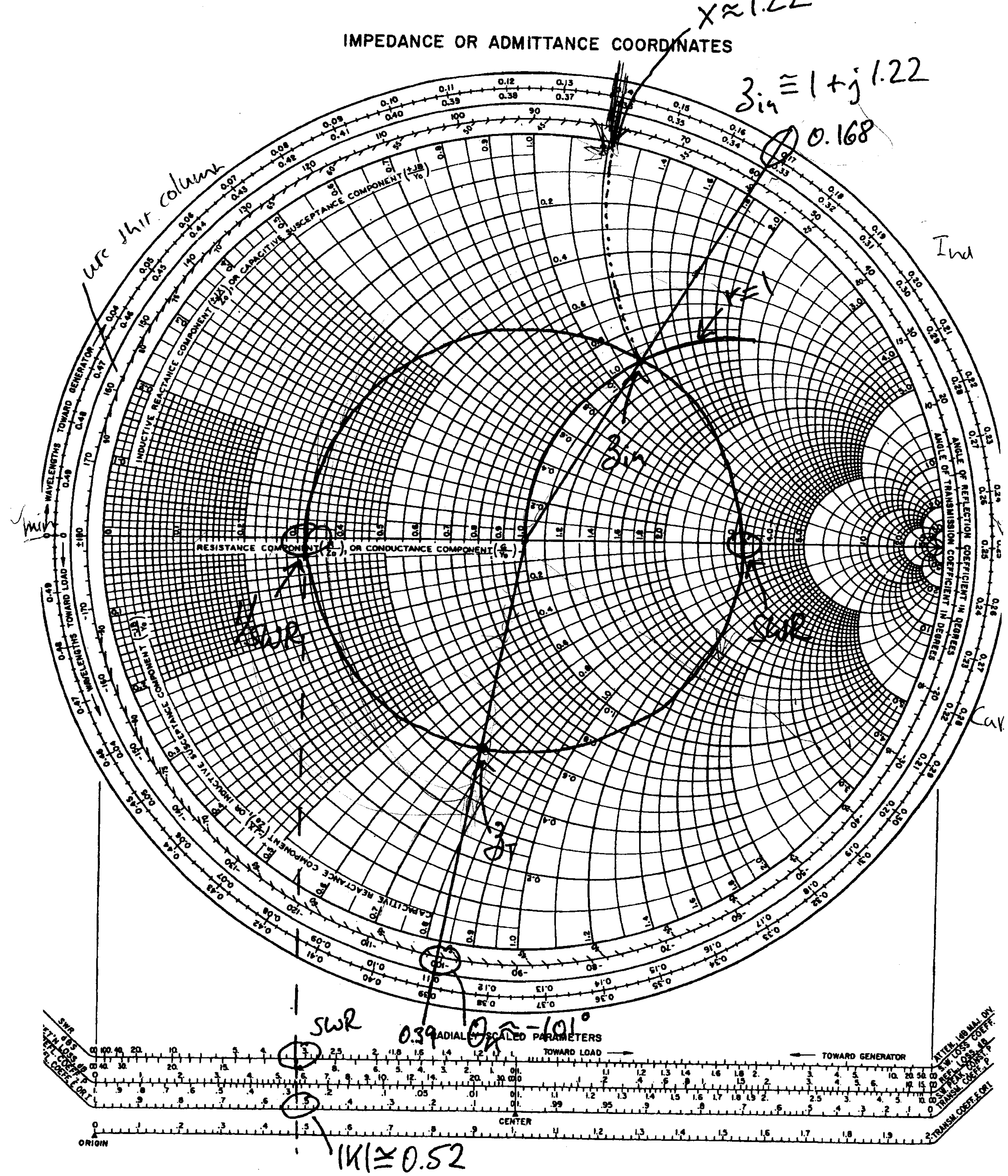
to compensate with a series-connected capacitor, we need to have

$$\frac{1}{j\omega C} = -j\,122\,\Omega$$

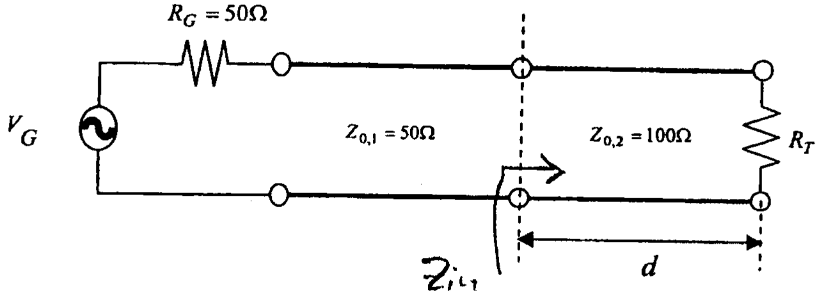
$$C = \frac{1}{\omega \cdot 122\,\Omega} \quad \left(= \frac{1}{\omega * 1.22 * Z_0} \right)$$

$$\cong \boxed{13\,pF}$$

Smith chart for problem 2



3. (10 pts.) An unknown resistive load R_T is connected through a transmission line section of length d and characteristic impedance $Z_{0,2} = 100\Omega$ to a $Z_{0,1} = 50\Omega$ transmission line, as shown below. At frequency f_0 the line length d corresponds to half a wavelength ($d = \lambda/2$).



(a) What is the length of the line if $f = f_0 = 200\,\text{MHz}$ and the effective dielectric constant of the transmission line is $\varepsilon_{r,eff} = 4$?

$$\lambda = \frac{v_p}{f} = \frac{c}{\sqrt{\varepsilon_{r,eff}}}\frac{1}{f} = 75cm$$

$$\Rightarrow d = \lambda/2 = \boxed{37.5cm}$$

(b) Determine R_T if at $f = f_0$ the standing-wave ratio on the 50Ω line is $SWR = 1$.

for $d = \lambda/2$, $Z_{in} = R_T$

Since $SWR = 1$, the line is matched $\Rightarrow R_T = Z_0 = \boxed{50\Omega}$

(c) What is the standing-wave ratio on the 50Ω line if the frequency is halved ($f = f_0/2$)?

for $f = f_0/2$, $d = \lambda/4$

$$Z_{in} = \frac{Z_{0,2}^2}{R_T} = \frac{(100\Omega)^2}{50\Omega} = 200\Omega = Z_{max} \text{ on line } 1$$

$$\Rightarrow SWR = \frac{200\Omega}{50\Omega} = \boxed{4}$$

(or $k = \frac{Z_{in} - Z_{0,1}}{Z_{in} + Z_{0,1}} = 3/5$ and $SWR = \frac{1+|k|}{1-|k|} = \frac{8/5}{2/5} = 4$)

(d) What is the standing-wave ratio on the 50Ω line at $f = f_0/2$ if the load resistor R_T is replaced with a capacitor $C = 31.8\,\text{pF}$ and the frequency is $f = f_0/2 = 100\,\text{MHz}$?

purely reactive (capacitive) termination

$$\Rightarrow \boxed{SWR = \infty}$$

ECE 391 – TRANSMISSION LINES AND ELECTROMAGNETIC WAVES

Winter Term 2002

Midterm II

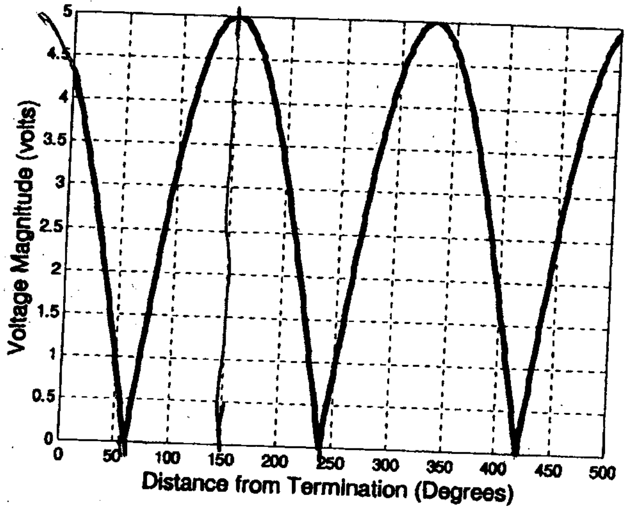
Name: Solutions (Last name, first name) SSN: xxx-xx-_________

Exam is closed book, closed notes; **one** sheet (2 pages) of formulas allowed; 50 minutes. Show all your work on the pages provided. No extra pages (use back if necessary). Read each question very carefully. Maximum number of points for each problem is given in parenthesis. Total: 40 points.

1. (10 pts.) A 100Ω transmission line is terminated in an **unknown** load impedance Z_T. The voltage standing-wave on the transmission line measured in electrical distance from the termination is shown below.

$\frac{\theta_K}{2} = -30^\circ$

$\Rightarrow \theta_K = -60^\circ$



$\frac{\theta_K}{2} = 150$

$\theta_K = 300^\circ$

$500 - 150 = 350$

2/2 (a) What is the standing-wave ratio on the line terminated in the unknown load impedance Z_T?

$$SWR = \frac{|V_{max}|}{|V_{min}|} = \frac{5V}{0V} = \boxed{\infty}$$

2/2 (b) Determine the voltage amplitude V_m^+ of the incident wave.

$$V_m^+ = \frac{|V_{max}| + |V_{min}|}{2} = \boxed{2.5V}$$

3/3 **(c) Determine the reflection coefficient at the termination (magnitude and phase).**

$$SWR = \infty \Rightarrow |K| = \frac{SWR-1}{SWR+1} = \boxed{1}$$

voltage minimum shifted from termination by

$$\frac{\theta_K}{2} + 90^\circ = 60^\circ \Rightarrow \boxed{\theta_K = -60^\circ}$$

$\phi = \theta_K - 2\beta z$

$\theta_K = \underset{60^\circ}{\underbrace{2\beta z}} \pm \pi$

Or find voltage maximum occurs at

$$\frac{\theta_K}{2} = 150^\circ \Rightarrow \boxed{\theta_K = 300^\circ \mathrel{\hat{=}} -60^\circ}$$

3/3 **(d) Determine the load impedance Z_T in ohms. (An <u>optional</u> Smith Chart is attached at the end.)**

from Smith chart: $z_T = -j1.73$

$$\Rightarrow Z_T = z_T * Z_0 = \boxed{-j\,173\,\Omega}$$

calculated:

$$Z_T = Z_0 \frac{1+K}{1-K} = Z_0 \frac{1+e^{-j60^\circ}}{1-e^{-j60^\circ}} = \boxed{-j\,173.2\,\Omega}$$

2. (18 pts.) A lossless transmission line of characteristic impedance $Z_0 = 100\Omega$ is terminated in an <u>unknown</u> complex impedance $Z_T = R_T + jX_T$. The normalized input impedance $z_{in}(d) = Z_{in}(d)/Z_0$ at distance $d = 1.2\lambda$ from the termination is specified on the attached Smith Chart. Use the attached Smith Chart to solve all of the following problems. **Show all your work on the Smith Chart!**

2/2 (a) Determine the input impedance $Z_{in}(d) = R + jX$ in ohms.

$$z_{in} = 0.5 - j0.6 = Z_{in}/Z_0$$

$$\Rightarrow \boxed{Z_{in} = (50 - j60)\,\Omega}$$

2/2 (b) Determine the corresponding input admittance in siemens.

from Smith chart

$$y_{in} \approx 0.82 + j0.98$$

$$\Rightarrow Y_{in} = \frac{y_{in}}{Z_0} = \boxed{(8.2 + j9.8) * 10^{-3}\,S}$$

4/4 (c) Determine the reflection coefficient (magnitude and phase in degrees) at distance d from the termination.

$$\left.\begin{array}{l} |K| \approx 0.48 \\ \theta_K \approx -108^\circ \end{array}\right\} \text{from Smith chart}$$

2/2 (d) Determine the voltage standing-wave ratio on the line.

$$VSWR \cong 2.85 \quad (\text{from Smith chart})$$

4/4 (e) Determine the terminating impedance $Z_T = R_T + jX_T$ in ohms.

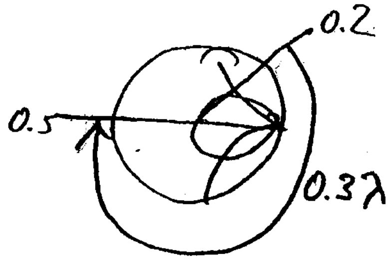
$$\frac{d}{\lambda} = 1.2 \triangleq 0.2 \Rightarrow \text{go from } 0.10 \text{ to } 0.30 \text{ (ccw)}$$

$$z_T \cong 1.7 + j1.25$$

$$\Rightarrow \boxed{Z_T = (170 + j125)\,\Omega}$$

2/2 (f) Determine the distance to the nearest voltage minimum $d_{v,min} / \lambda$ from the termination.

$$\frac{d_{v,min}}{\lambda} = 0.5 - 0.2 = \boxed{0.3}$$



2/2 (g) Now, assume that the operating frequency is increased by 20% and the terminating impedance remains unchanged. Determine the new input impedance at distance d from the termination. Hint: the <u>physical</u> distance d remains unchanged.

$f_{new} = f_{old} * 1.2$ (20% increase)

$\Rightarrow \lambda_{new} = \lambda_{old}/1.2 = 0.8333\,\lambda_{old}$ $\quad \frac{1}{1.2} = .8333$

$$\frac{d}{\lambda_{new}} = \frac{d}{\lambda_{old}} \cdot 1.2 = 1.2 * 1.2 = 1.44 \hat{=} 0.44$$

$\Rightarrow$ rotate from z_T clockwise by 0.44

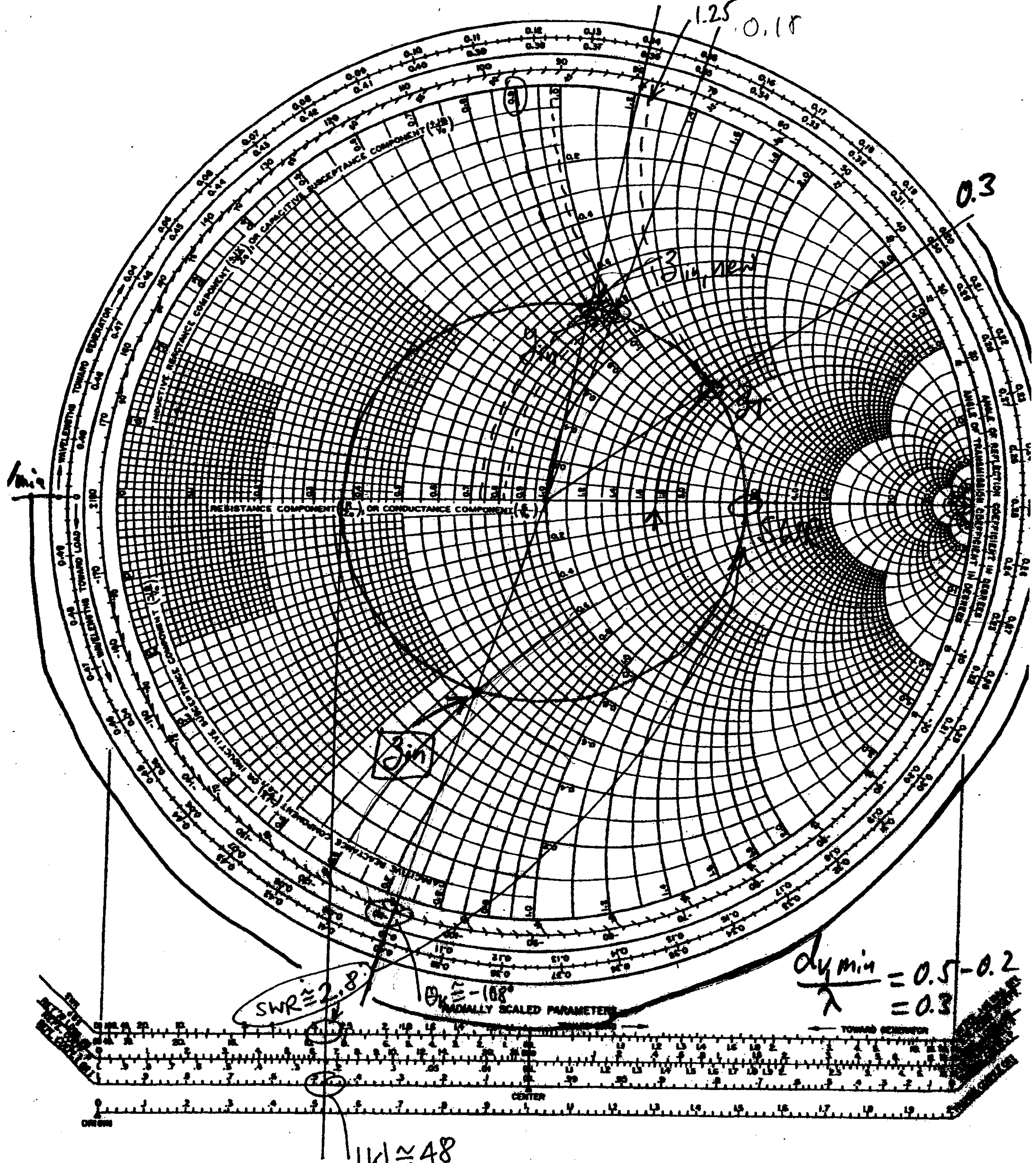
$0.44 + 0.2 = 0.64 \hat{=} 0.14$

$z_{in,new} \hat{=} 0.73 + j0.9$

$$\boxed{Z_{in,new} \hat{=} (73 + j90)\,\Omega}$$

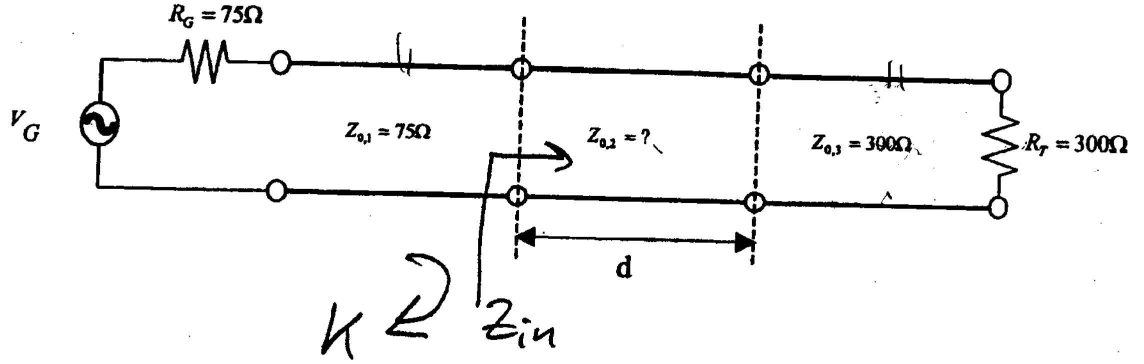
Problem #2

IMPEDANCE OR ADMITTANCE COORDINATES



$$\frac{d_{y\,min}}{\lambda} = 0.5 - 0.2 = 0.3$$

3. (12 pts.) A 300Ω load is to be matched to a 75Ω lossless coaxial cable through a section of transmission line of undetermined characteristic impedance $Z_{0,2}$ and length d, as shown below.



3/3 (a) Determine the characteristic impedance $Z_{0,2}$ of the connecting line section.

use quarter-wave transformer to match

$$Z_{0,2} = \sqrt{Z_{0,1} * Z_{0,3}} = \sqrt{75\Omega * 300\Omega} = \boxed{150\Omega}$$

3/3 (b) For an operating frequency of $f = 125\,\text{MHz}$, determine the shortest length d of the connecting line section to achieve a match. Assume a propagation velocity of $v_p = 25\,\text{cm/nsec}$.

$$\lambda = \frac{V_p}{f} = \frac{2.5\times10^8 \frac{m}{s}}{125\times10^6 \frac{1}{s}} = 2m$$

$$d = \frac{\lambda}{4} = \boxed{50cm} \quad \text{then, } Z_{in} = \boxed{75\Omega}$$

$\frac{150^2}{300}$

3/3 (c) What is the standing-wave ratio on the line connected to the generator if the operating frequency is doubled ($f = 250$ MHz)?

$\frac{\lambda}{4} + \frac{\lambda}{4}$ $\quad \frac{\lambda}{2}$

for $f = 250\,MHz$ $\lambda = 1m$ $\quad Z_{in} = \frac{Z_0 Z_T}{Z_0}$ $\quad \frac{300}{} \quad = 300$

$$\Rightarrow \frac{d}{\lambda} = \frac{1}{2} \Rightarrow \text{half-wave transformer}$$

$$\Rightarrow Z_{in} = \boxed{300\Omega}$$

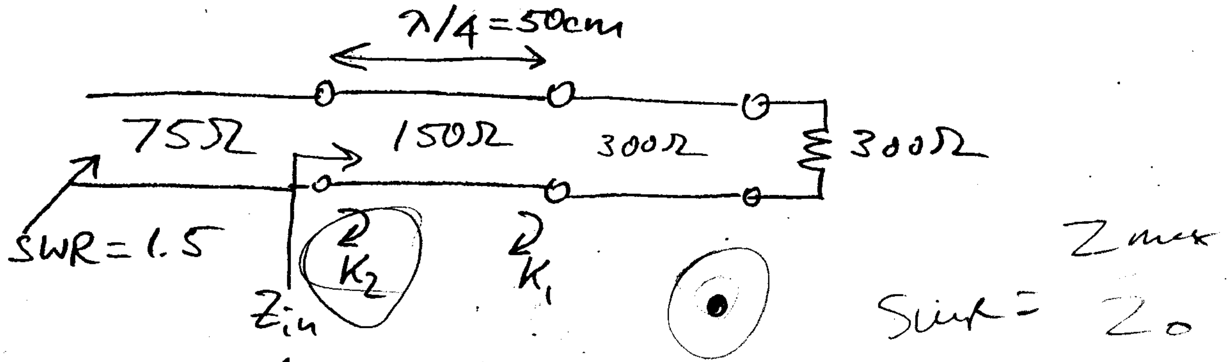
$$|K| = \frac{Z_{in} - Z_{0,1}}{Z_{in} + Z_{0,1}} = \frac{225}{375} = 0.6$$

$$SWR = \frac{1+|K|}{1-|K|} = \frac{1.6}{0.4} = \boxed{4}$$

or

$$SWR = \frac{Z_{max}}{Z_0} = \frac{300\Omega}{75\Omega} = \boxed{4}$$

3/3 (d) Now the connecting line with characteristic impedance $Z_{0,2}$ is lossy, resulting in a mismatch on the line connected to the generator at $f = 125$ MHz. The standing-wave ratio on the line connected to the generator is found as $SWR = 1.5$. Determine the attenuation constant of the connecting lossy line. (An optional Smith chart is attached at the end.) Note: You may want to work on this problem last.

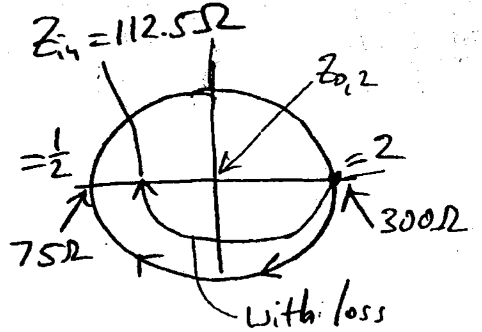


Without loss: $Z_{in} = 75\Omega$

With loss in λ/4 section: $Z_{in} > 75\Omega$ (closer to $Z_{0,2} = 150\Omega$)

$\Rightarrow$ from $SWR = 1.5$ follows $Z_{in} = Z_{max} = SWR \cdot Z_{0,1}$

$= 112.5\Omega$



W/o loss: $|K_1| = |K_2|$

With loss $|K_2| = |K_1| e^{-2\alpha d}$

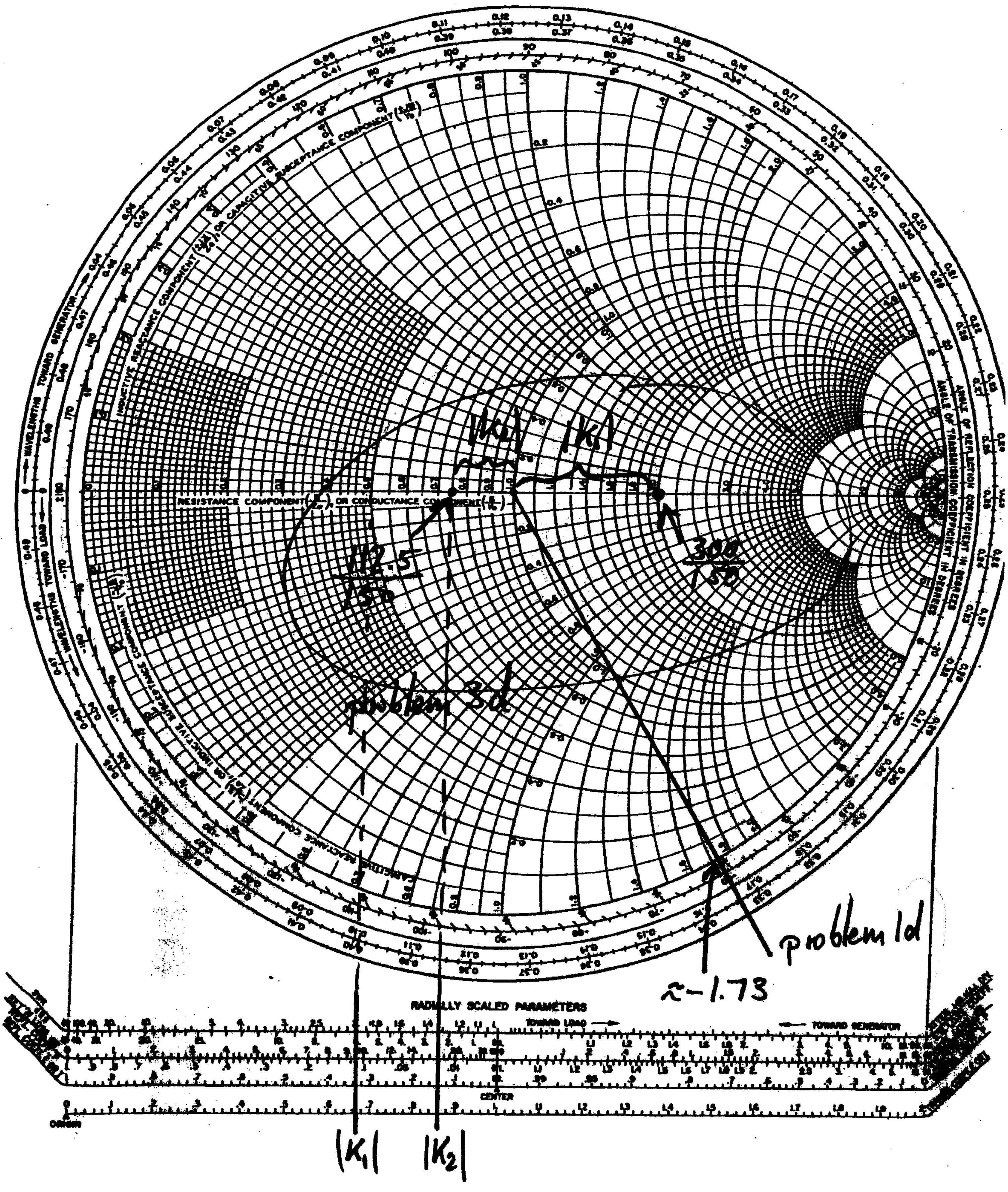
calculated or from Smith chart:

$$\left.\begin{aligned} |K_1| &= \left|\frac{300-150}{300+150}\right| = \frac{1}{3} \\ |K_2| &= \left|\frac{112.5-150}{112.5+150}\right| = 0.143 \end{aligned}\right\} \Rightarrow \alpha = \frac{1}{2d} \ln \frac{|K_1|}{|K_2|}$$

$$\boxed{= 0.845 \frac{Np}{m}}$$

Optional Smith chart for problems #1 and #3

IMPEDANCE OR ADMITTANCE COORDINATES



Stub matching, The new and improved Zane method.

RULES:
LENGTH and DISTANCE of stub must be positive.
Distance in the direction of the generator is always positive (clockwise).
Always measure distance going in the direction of the generator.
Distance must be measured in lambda's.
Always use outer most ring on chart when measuring lambda.
Smiling is a must when solving.

PROCEDURE:
1. normalize
2. plot on smith chart
3. draw radial load line
4. draw K= circle
5. if shunt, change to admittance
6. if shunt, draw new load line
7. locate intersections of r=1 (g=1) circle and k= circle
8a. for inductive solution choose point in capacitive region,
8b. for capacitive solution choose point in inductive region.
9. draw stub line from 1+j0 to chosen point
10. measure distance in lambda from the load line to the stub line(clockwise direction)
11. record distance as the distance from load to stub
12. find imaginary part of the stub line at r=1 (g=1)
13. plot the inverse of the imaginary part on the g=0 circle(should be on other side of horizontal axis)
14. this new line is the length line
15. .The following is how to get the length of the stub
15a. for open stub in shunt configuration: measure distance from left side to the length line
15b. for shorted stub in shunt configuration: measure distance from right side to the length line
15c. for open stub in series configuration: measure distance from right side to the length line
15d. for shorted stub in series configuration: measure distance from left side. to the length line

References:
http://classes.engr.orst.edu/eecs/winter04/ece391
http://www.ee.surrey.ac.uk/Personal/D.Jefferies/jefferies-stub.html